Assignment 3

is on website

Due Thurs Mar 12, 8:30 am

Test 3 (4 Questions)

Fri March 20

Covers 4.6, 4.7, 5.1, 6.2

6.2 Series Solutions Cont'd

Ex: Solve $y'' - xy = 0$

using $y = \sum_{n=0}^{\infty} c_n x^n$.

Find the first 3 nonzero terms of y_1 and y_2.

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"Recurrence Relation"

$$c_2 = 0$$

$$c_{k+2} = \frac{c_{k-1}}{(k+2)(k+1)} \quad \text{for } k \geq 1$$

Generate Coefficients

C_0 unknown

C_1 unknown

$C_2 = 0$

$(k=1)$ $\quad C_3 = \frac{C_0}{3 \cdot 2} = \frac{C_0}{6}$

$(k=2)$ $\quad C_4 = \frac{C_1}{4 \cdot 3} = \frac{C_1}{12}$

$(k=3)$ $\quad C_5 = \frac{C_2}{5 \cdot 4} = 0$

$(k=4)$ $\quad C_6 = \frac{C_3}{6 \cdot 5} = \frac{1}{30}\left(\frac{C_0}{6}\right) = \frac{C_0}{180}$

$(k=5)$ $\quad C_7 = \frac{C_4}{7 \cdot 6} = \frac{1}{42}\left(\frac{C_1}{12}\right) = \frac{C_1}{504}$

Recall $y = \sum_{n=0}^{\infty} C_n x^n$

$y = C_0 + C_1 x + C_2 x^2 + \cdots$

$$y = C_0 + C_1 x + \frac{C_0}{6}x^3 + \frac{C_1}{12}x^4 + \frac{C_0}{180}x^6 + \frac{C_1}{504}x^7 + \cdots$$

$$y = C_0\underbrace{\left[1 + \frac{x^3}{6} + \frac{x^6}{180} + \cdots\right]}_{y_1} + C_1\underbrace{\left[x + \frac{x^4}{12} + \frac{x^7}{504} + \cdots\right]}_{y_2}$$

Notice: We get 2 solutions, y_1 and y_2, because it's a 2nd-order DE.

Ex: Solve $y'' - y = 0$, $y(0) = 3$, $y'(0) = 5$, using $y = \sum_{n=0}^{\infty} C_n x^n$.

Find the first six nonzero terms.

$$y = \sum_{n=0}^{\infty} C_n x^n$$

$$y' = \sum_{n=1}^{\infty} n C_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) C_n x^{n-2}$$

$$y'' - y = 0$$

$$\sum_{n=2}^{\infty} n(n-1) C_n x^{n-2} - \sum_{n=0}^{\infty} C_n x^n = 0$$

$k = n-2$

$n = k+2$

$n = 2 \Rightarrow k = 0$

$k = n$

$n = 0 \Rightarrow k = 0$

Rewrite using k.
All series start at the largest k-value (k=0)

$$\sum_{k=0}^{\infty} (k+2)(k+1) C_{k+2} x^k - \sum_{k=0}^{\infty} C_k x^k = 0$$

$$\sum_{k=0}^{\infty} \left[(k+2)(k+1)\, C_{k+2} - C_k \right] x^k = 0$$

Set coefficients = 0

$$(k+2)(k+1)\, C_{k+2} - C_k = 0$$

$$C_{k+2} = \frac{C_k}{(k+2)(k+1)}, \quad k \geq 0$$

"recurrence relation"

Use initial conditions.

$$y = \sum_{n=0}^{\infty} C_n x^n$$

$$y = C_0 + C_1 x + C_2 x^2 + \cdots$$

$y\big|_{x=0} = 3$: $\quad 3 = C_0$

$$y' = C_1 + 2C_2 x + \cdots$$

$y'\big|_{x=0} = 5$: $\quad 5 = C_1$

Generate coefficients

$(k=0)$ $\quad C_2 = \frac{C_0}{2 \cdot 1} = \frac{3}{2}$

$(k=1)$ $\quad c_3 = \frac{c_1}{3\cdot 2} = \frac{5}{6}$

$(k=2)$ $\quad c_4 = \frac{c_2}{4\cdot 3} = \frac{1}{12}\left(\frac{3}{2}\right) = \frac{1}{8}$

$(k=3)$ $\quad c_5 = \frac{c_3}{5\cdot 4} = \frac{1}{20}\left(\frac{5}{6}\right) = \frac{1}{24}$

$$y = c_0 + c_1 x + c_2 x^2 + \dots$$

$$y = 3 + 5x + \frac{3}{2}x^2 + \frac{5}{6}x^3 + \frac{1}{8}x^4 + \frac{1}{24}x^5 + \dots$$

Ex: Find the first 3 nonzero terms of y_1 and y_2 using $y = \sum_{n=0}^{\infty} c_n x^n$:

$$y'' - (1+x)y = 0$$

$$y = \sum_{n=0}^{\infty} c_n x^n$$

$$y' = \sum_{n=1}^{\infty} n c_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$$

$$y'' - (1+x)y = 0$$

$$\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} - (1+x)\sum_{n=0}^{\infty} c_n x^n = 0 \qquad +(-1-x)$$

$$\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} - \sum_{n=0}^{\infty} c_n x^n - x\sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} - \sum_{n=0}^{\infty} c_n x^n - \sum_{n=0}^{\infty} c_n x^{n+1} = 0$$

$k = n-2$	$k = n$	$k = n+1$
$n = k+2$	$n=0 \Rightarrow k=0$	$n = k-1$
$n=2 \Rightarrow k=0$		$n=0 \Rightarrow k=1$

Rewrite using k.

All series start at largest k-value ($k=1$).

$$\boxed{k=0 \text{ term}} + \sum_{k=1}^{\infty} \sim \quad - \boxed{k=0 \text{ term}} - \sum_{k=1}^{\infty} \sim - \sum_{k=1}^{\infty} \sim = 0$$

$$2c_2 x^0 + \sum_{k=1}^{\infty} (k+2)(k+1)c_{k+2} x^k - c_0 x^0$$

$$- \sum_{k=1}^{\infty} c_k x^k - \sum_{k=1}^{\infty} c_{k-1} x^k = 0$$

$$(2C_2 - C_0) + \sum_{k=1}^{\infty} \left[(k+2)(k+1)C_{k+2} - C_k - C_{k-1} \right] x^k = 0$$