## Assignment 2

is on website

Due Mon Feb 23, 8:30am

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## Test 2

Fri Feb 27

Covers 3.1, 4.1–4.4

The question from Section 3.1 will be "Set up the DE; Don't solve the DE."

## 4.6 Variation of Parameters Cont'd

Ex: Solve $3y'' + 15y' + 18y = 36\sec e^x$

$\vdots$

$y_1 = e^{-2x}$, $y_2 = e^{-3x}$

$\vdots$

$u_1 = e^x \sin e^x + 6\sec e^x$

$u_2 = -e^{2x} \sin e^x - 2e^x 6\sec e^x + 2\sin e^x$

5) Get $y_p$

$y_p = u_1 y_1 + u_2 y_2$

$$= (e^x \sin e^x + \cos e^x)\, e^{-2x}$$

$$+ (-e^{2x} \sin e^x - 2e^x \cos e^x + 2\sin e^x)\, e^{-3x}$$

$$= e^{-x} \sin e^x + e^{-2x} \cos e^x$$

$$- e^{-x} \sin e^x - 2e^{-2x} \cos e^x + 2e^{-3x} \sin e^x$$

$$= -e^{-2x} \cos e^x + 2e^{-3x} \sin e^x$$

6) Get $y$

$$y = y_c + y_p$$

$$y = C_1 e^{-2x} + C_2 e^{-3x} - e^{-2x} \cos e^x + 2e^{-3x} \sin e^x$$

7) ~~Initial Conditions~~

Ex: Solve $y'' - 3y' + 2y = x^3$ using Variation of Parameters.

Note: Could also use Section 4.4 (Method of Undetermined Coefficients) with $y_p = Ax^3 + Bx^2 + Cx + D$

1) $y_c$

$$y'' - 3y' + 2y = 0$$

$$m^2 - 3m + 2 = 0$$

$$(m-1)(m-2)=0$$

$$m=1,2$$

$$y_1=e^{x}\quad,\quad y_2=e^{2x}$$

$$y_c=C_1e^{x}+C_2e^{2x}$$

2) $W,\ W_1,\ W_2$

$$W=\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$=\begin{vmatrix} e^{x} & e^{2x} \\ e^{x} & 2e^{2x} \end{vmatrix}$$

$$=2e^{3x}-e^{3x}$$

$$=e^{3x}$$

$$W_1=\begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix}$$

Caution: Use standard form of DE. ✓

$$=\begin{vmatrix} 0 & e^{2x} \\ x^3 & \sim \end{vmatrix}$$

$$=-x^3e^{2x}$$

$$W_2=\begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix}$$

Caution: Use standard form of DE. ✓

$$= \begin{vmatrix} e^{x} & 0 \\ \sim & x^3 \end{vmatrix}$$

$$= x^3 e^{x}$$

3) Get $u_1$

$$u_1' = \frac{W_1}{W}$$

$$= \frac{-x^3 e^{2x}}{e^{3x}}$$

$$= -x^3 e^{-x}$$

$$u_1 = \int -x^3 e^{-x}\, dx$$

| | D | I |
|---|---|---|
| ⊕ | $-x^3$ | $e^{-x}$ |
| ⊖ | $-3x^2$ | $-e^{-x}$ |
| ⊕ | $-6x$ | $e^{-x}$ |
| ⊖ | $-6$ | $-e^{-x}$ |
| | $0$ | $e^{-x}$ |

$$u_1 = (x^3 + 3x^2 + 6x + 6)e^{-x}$$

Don't use a constant here.

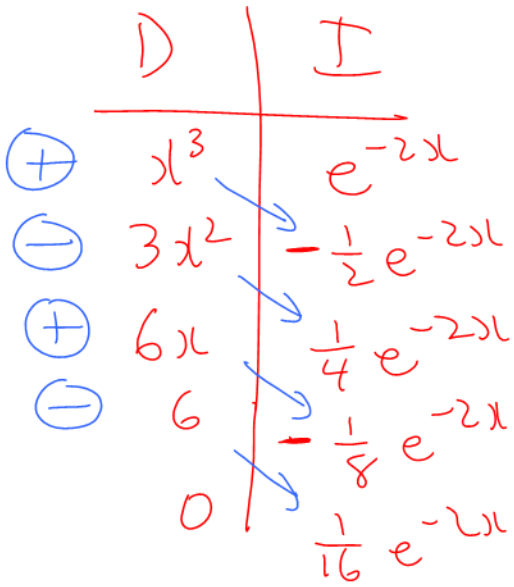
4) Get $u_2$

$$u_2' = \frac{W_2}{W}$$

$$= \frac{x^3 e^x}{e^{3x}}$$

$$= x^3 e^{-2x}$$

$$u_2 = \int x^3 e^{-2x}\,dx$$




$$u_2 = \left(-\frac{x^3}{2} - \frac{3x^2}{4} - \frac{6x}{8} - \frac{6}{16}\right)e^{-2x}$$

5) Get $y_p$

$$y_p = u_1 y_1 + u_2 y_2$$

$$= (x^3 + 3x^2 + 6x + 6)e^{-x} \cdot e^{x}$$

$$+ \left(-\frac{x^3}{2} - \frac{3x^2}{4} - \frac{3x}{4} - \frac{3}{8}\right)e^{-2x} \cdot e^{2x}$$

$$= \frac{x^3}{2} + \frac{9x^2}{4} + \frac{21x}{4} + \frac{45}{8}$$

6) Get $y$

$$y = y_c + y_p$$

$$y = C_1 e^{x} + C_2 e^{2x} + \frac{x^3}{2} + \frac{9x^2}{4} + \frac{21x}{4} + \frac{45}{8}$$

~~7) Initial Conditions~~

## 4.7 Cauchy-Euler DEs

Consider $x^2 y'' - 7xy' + 15y = 0$

Power of $x$ matches the order of the derivative (on each term).

"Cauchy-Euler DE"

FACT

Auxiliary equation for Cauchy-Euler DEs:

$x^3 y''' \rightarrow m(m-1)(m-2)$

$x^2 y'' \rightarrow m(m-1)$

$xy' \rightarrow m$

$y \rightarrow 1$

Above: $m(m-1) - 7m + 15 = 0$

$m^2 - m - 7m + 15 = 0$

$$m^2 - 8m + 15 = 0$$

FACT

| Roots of Auxiliary Equation | Solutions to DE |
|---|---|
| Distinct Real #<br>$m_1, m_2, m_3, \ldots$ | $x^{m_1}, x^{m_2}, x^{m_3}, \ldots$ |
| Repeated Real Roots<br>$m, m, m, \ldots$ | $x^m, x^m \ln x, x^m (\ln x)^2, \ldots$ |
| Complex Roots<br>$\alpha \pm \beta i$ | $x^\alpha [C_1 \cos(\beta \ln x) + C_2 \sin(\beta \ln x)]$ |

Ex: Solve $x^2 y'' - 7xy' + 15y = 0$

Cauchy-Euler DE

$$m(m-1) - 7m + 15 = 0$$

$$m^2 - 8m + 15 = 0$$

$$(m-3)(m-5) = 0$$

$$m = 3, 5$$

$$y_1 = x^3 \quad , y_2 = x^5$$

$$y = C_1 x^3 + C_2 x^5$$