Assignment 2

Is on website

Due Mon Feb 23, 8:30am

Submit to D2L Dropbox

Test 2 (5 Questions)

Fri Feb 27

Covers 3.1, 4.1-4.4

The question from Section 3.1 will be "Set up a DE; Don't solve the DE."

4.4 Nonhomogeneous DEs with Constant Coefficients Cont'd

Ex: Solve $y'' - 6y' + 8y = 3e^{2x}$,

$y(0) = 0, \quad y'(0) = 0.$

1) Find y_c

$$m^2 - 6m + 8 = 0$$

$$(m-2)(m-4) = 0$$

$$m = 2, 4 \Rightarrow y_1 = e^{2x}, \quad y_2 = e^{4x}$$

$$y_c = C_1 e^{2x} + C_2 e^{4x}$$

2) Find y_p

$f(x) = 3e^{2x}$

~~$y_p = Ae^{2x}$~~ (bad case)

$y_p = Axe^{2x}$

3) $y_p \to DE$ (Find A)

$y_p = Axe^{2x}$

$y_p' = 2Axe^{2x} + Ae^{2x}$

$y_p'' = 4Axe^{2x} + 2Ae^{2x} + 2Ae^{2x}$

(product rule)

$y_p'' = 4Axe^{2x} + 4Ae^{2x}$

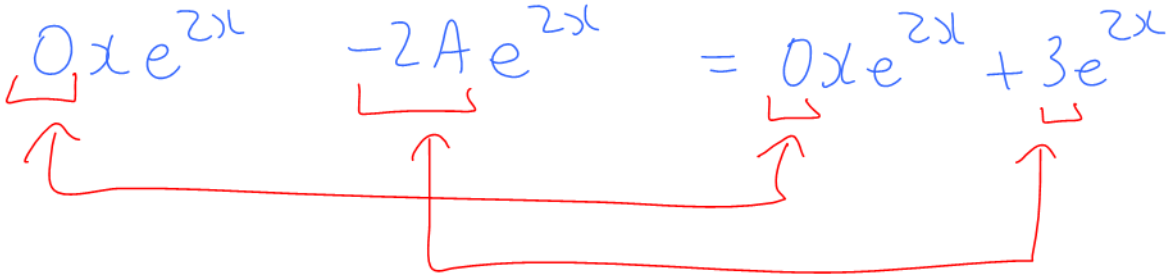
$y'' - 6y' + 8y = 3e^{2x}$

$(4Axe^{2x} + 4Ae^{2x}) - 6(2Axe^{2x} + Ae^{2x})$

$+ 8Axe^{2x} = 3e^{2x}$

Match coefficients:

$(4A - 12A + 8A)xe^{2x} + (4A - 6A)e^{2x} = 0xe^{2x} + 3e^{2x}$



$$-2A = 3$$

$$A = \frac{-3}{2}$$

$$y_p = -\frac{3}{2} x e^{2x}$$

4) $y = y_c + y_p$

$$y = C_1 e^{2x} + C_2 e^{4x} - \frac{3}{2} x e^{2x}$$

5) Initial Conditions

$y = 0$, $x = 0$: $0 = C_1 + C_2$ ①

$$y' = 2C_1 e^{2x} + 4C_2 e^{4x} - \frac{3}{2}(2x e^{2x} + e^{2x})$$

$y' = 0$, $x = 0$: $0 = 2C_1 + 4C_2 - \frac{3}{2}$

$$2C_1 + 4C_2 = \frac{3}{2}$$ ②

$-2 \times$ ① : $-2C_1 - 2C_2 = 0$

② : $+\ 2C_1 + 4C_2 = \frac{3}{2}$

$$2C_2 = \frac{3}{2}$$

$$C_2 = \frac{3}{4}$$

$C_2 = \frac{3}{4} \to (1)$: $\quad 0 = C_1 + \frac{3}{4}$

$$C_1 = -\frac{3}{4}$$

$$y = -\frac{3}{4}e^{2x} + \frac{3}{4}e^{4x} - \frac{3}{2}xe^{2x}$$

Ex: Solve $y'' - y = xe^{4x}$

1) Find y_c

$$m^2 - 1 = 0$$

$$(m-1)(m+1) = 0$$

$$m = \pm 1$$

$$y_1 = e^{x}, \quad y_2 = e^{-x}$$

$$y_c = C_1 e^{x} + C_2 e^{-x}$$

2) Find y_p

$$f(x) = xe^{4x}$$

$$f'(x) = 4xe^{4x} + e^{4x}$$

$$f''(x) = 16xe^{4x} + 4e^{4x} + 4e^{4x}$$

no new like terms

$$y_p = Axe^{4x} + Be^{4x}$$

Bad case? No.

3) $y_p \to DE$ (Find A, B)

$$y_p = Axe^{4x} + Be^{4x}$$
$$y_p' = 4Axe^{4x} + Ae^{4x} + 4Be^{4x}$$
$$y_p'' = 16Axe^{4x} + 4Ae^{4x} + 4Ae^{4x} + 16Be^{4x}$$

product rule

$$\boxed{y_p'' = 16Axe^{4x} + 8Ae^{4x} + 16Be^{4x}}$$

$$y'' - y = xe^{4x}$$

$$16Axe^{4x} + 8Ae^{4x} + 16Be^{4x} - (Axe^{4x} + Be^{4x}) = xe^{4x}$$

$$\underbrace{(16A - A)}_{15A} xe^{4x} + \underbrace{(8A + 16B - B)}_{(8A+15B)} e^{4x} = 1xe^{4x} + 0e^{4x}$$

$$15A = 1 \quad (1)$$
$$8A + 15B = 0 \quad (2)$$

① : $A = \frac{1}{15}$

$A = \frac{1}{15} \rightarrow$ ② : $\frac{8}{15} + 15B = 0$

$15B = \frac{-8}{15}$

$B = \frac{-8}{225}$

$$y_p = \frac{1}{15} x e^{4x} - \frac{8}{225} e^{4x}$$

4) $y = y_c + y_p$

$$y = C_1 e^{x} + C_2 e^{-x} + \frac{1}{15} x e^{4x} - \frac{8}{225} e^{4x}$$

~~5) Initial Conditions~~

The method of this section is called "Undetermined Coefficients"