Assignment 2

Is on website
Due Mon Feb 23, 8:30 am
Submit to D2L Dropbox

Test 2 (5 Questions)

Fri Feb 27
Covers 3.1, 4.1 - 4.4

4.4 Nonhomogeneous DEs with Constant Coefficients Cont'd

FACT

y_p Consists of all terms in $f(x)$, $f'(x)$, $f''(x)$, ..., and it has unknown coefficients

$f(x)$	y_p
polynomial of degree n	$A_n x^n + \dots + A_1 x + A_0$
$\#\cos kx + \#\sin kx$	$A\cos kx + B\sin kx$
$\#e^{kx}$	Ae^{kx}

Ex: Find y_p

a) $f(x) = 4xe^{6x}$

$f'(x) = 24xe^{6x} + 4e^{6x}$

$f''(x) = 144xe^{6x} + 24e^{6x} + 24e^{6x}$

no new like terms

$y_p = Axe^{6x} + Be^{6x}$

b) $f(x) = e^{2x}\sin 7x$

$f'(x) = 7e^{2x}\cos 7x + 2e^{2x}\sin 7x$

$f''(x) = -49e^{2x}\sin 7x + 14e^{2x}\cos 7x$

$+ 14e^{2x}\cos 7x + 4e^{2x}\sin 7x$

no new like terms

$y_p = Ae^{2x}\cos 7x + Be^{2x}\sin 7x$

Ex: Solve $y'' - 4y' + 4y = 2\sin 3x$

1) Find y_c

Solve $y'' - 4y' + 4y = 0$

$m^2 - 4m + 4 = 0$

$$(m-2)^2 = 0$$

$$m = 2, 2$$

$$y_1 = e^{2x} \quad , \quad y_2 = xe^{2x}$$

$$y_c = C_1 e^{2x} + C_2 x e^{2x}$$

2) Find y_p

$$f(x) = 2\sin 3x$$

$$y_p = A\cos 3x + B\sin 3x$$

3) $y_p \rightarrow DE$ (Find A, B)

$$\begin{cases} y_p = A\cos 3x + B\sin 3x \\ y_p' = -3A\sin 3x + 3B\cos 3x \\ y_p'' = -9A\cos 3x - 9B\sin 3x \end{cases}$$

$$y'' - 4y' + 4y = 2\sin 3x$$

$$-9A\cos 3x - 9B\sin 3x - 4(-3A\sin 3x + 3B\cos 3x)$$
$$+ 4(A\cos 3x + B\sin 3x) = 2\sin 3x$$

$$\boxed{-9A - 12B + 4A}\cos 3x + \boxed{-9B + 12A + 4B}\sin 3x = 0\cos 3x + 2\sin 3x$$

Match Coefficients:

$$-5A - 12B = 0 \quad (1)$$

$$12A - 5B = 2 \quad (2)$$

$12 \times (1)$: $-60A - 144B = 0$

$5 \times (2)$: $60A - 25B = 10$

$$+ \quad \overline{\qquad\qquad\qquad\qquad}$$

$$-169B = 10 \Rightarrow B = \frac{-10}{169}$$

$B = \frac{-10}{169} \rightarrow (1)$: $-5A + \frac{120}{169} = 0$

$$-5A = \frac{-120}{169}$$

$$A = \frac{24}{169}$$

$$y_p = A\cos 3x + B\sin 3x$$

$$y_p = \frac{24}{169}\cos 3x - \frac{10}{169}\sin 3x$$

4) $y = y_c + y_p$

$$y = C_1 e^{2x} + C_2 x e^{2x} + \frac{24}{169}\cos 3x - \frac{10}{169}\sin 3x$$

5) ~~Initial Conditions~~

The Bad Case

Any terms in y_p that also appear in y_c must be multiplied by the smallest power of x that eliminates the duplication.

Ex: Given $y_c = C_1 e^x + C_2 x e^x$.

Find y_p if:

a) $f(x) = 5e^x + 6e^{2x}$

$y_p = Ae^x + Be^{2x}$?

Ae^x ← duplication

Be^{2x} ← no duplication

$y_p = Ax^2 e^x + Be^{2x}$

b) $f(x) = e^x \cos x + 1$

$f'(x) = -e^x \sin x + e^x \cos x$

$f''(x) = -e^x \cos x - e^x \sin x - e^x \sin x + e^x \cos x$

no new like terms

$y_p = Ae^x \cos x + Be^x \sin x + C$

(not bad case)

c) $f(x) = 7xe^x$

$f'(x) = 7xe^x + 7e^x$

$f''(x) = 7xe^x + 7e^x + 7e^x$

no new like terms

$y_p = Axe^x + Be^x$?

$Be^x \rightarrow Bx^2e^x$

$Axe^x \rightarrow Ax^3e^x$

$y_p = Ax^3e^x + Bx^2e^x$