Test Average = 72%

If (exam grade) > (calculated grade)
then (final grade in course) = (exam grade).

Test 2
~~Thurs Feb 26~~ → Fri ~~Feb~~ 27

Test 3
~~Thurs Mar 19~~ → Fri Mar 20

4.2 Finding a Second Solution

Standard form for a homogeneous 2nd-order linear DE:

$$y'' + P(x)\,y' + Q(x)\,y = 0$$

Given one solution y_1,
a second linearly-independent solution is:

$$y_2 = y_1 \int \frac{e^{-\int P(x)\,dx}}{y_1^2}\,dx$$

(see formula sheet)

General solution to DE:

$$y = C_1 y_1 + C_2 y_2$$

Ex: $y'' = 6y' - 9y$

a) Check that $y_1 = e^{3x}$ is a solution to the DE.

$y_1 = e^{3x}$

$y_1' = 3e^{3x}$

$y_1'' = 9e^{3x}$

$LS = y''$

$= 9e^{3x}$

$RS = 6y' - 9y$

$= 6(3e^{3x}) - 9(e^{3x})$

$= 18e^{3x} - 9e^{3x}$

$= 9e^{3x}$

b) Find a second linearly independent solution to the DE.

DE: $y'' - 6y' + 9y = 0$ standard form

$P(x) = -6$

$\int P(x)dx = -6x$ ← don't use a constant here

$-\int P(x)dx = 6x$

$$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2}\, dx$$

$$= e^{3x} \int \frac{e^{6x}}{(e^{3x})^2}\, dx$$

$$= e^{3x} \int \frac{e^{6x}}{e^{6x}}\, dx$$

$$= e^{3x} \int 1\, dx$$

$$= e^{3x}(x) \quad \leftarrow \text{don't use a constant here}$$

$$= xe^{3x}$$

c) General solution to the DE?

$$y = C_1 y_1 + C_2 y_2$$

$$y = C_1 e^{3x} + C_2 x e^{3x}$$

Ex: $y'' - 12y' - 493y = 0$ has solution $y_1 = e^{29x}$.

a) Find a second linearly independent solution to the DE.

Standard Form ✓

$P(x) = -12$

$\int P(x)\, dx = -12x$

$$-\int P(x)\,dx = 12x$$

$$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2}\,dx$$

$$= e^{29x} \int \frac{e^{12x}}{(e^{29x})^2}\,dx$$

$$= e^{29x} \int \frac{e^{12x}}{e^{58x}}\,dx$$

$$= e^{29x} \int e^{-46x}\,dx$$

$$= e^{29x} \left(\frac{e^{-46x}}{-46} \right)$$

$$= -\frac{1}{46} e^{-17x}$$ ✓

or $y_2 = e^{-17x}$ ✓

(Because $y = C_1 y_1 + C_2 y_2$)

b) General Solution to the DE?

$$y = C_1 y_1 + C_2 y_2$$

$$y = C_1 e^{29x} + C_2 e^{-17x}$$

Ex: $y'' + 25y = 0$

has solution $y_1 = \sin 5x$.

a) Find a second linearly independent solution.

Standard Form ✓

$P(x) = 0$

$\int P(x)\,dx = 0$ ← don't we a constant here

$-\int P(x)\,dx = 0$

$$y_2 = y_1 \int \frac{e^{-\int P(x)\,dx}}{y_1^2}\,dx$$

$$= \sin 5x \int \frac{e^0}{(\sin 5x)^2}\,dx \qquad e^0 = 1$$

$$= \sin 5x \int \csc^2 5x\,dx$$

$$= \sin 5x \left(-\frac{\cot 5x}{5}\right)$$

$$= \sin 5x \left(-\frac{1}{5}\frac{\cos 5x}{\sin 5x}\right)$$

$$= -\frac{1}{5}\cos 5x \quad ✓$$

or $y_2 = \cos 5x$ ✓

b) General Solution to DE?

$$y = C_1 y_1 + C_2 y_2$$

$$y = C_1 \sin 5x + C_2 \cos 5x$$