Test Covers 1.1–1.2, 2.2–2.5

Ex: $\frac{dy}{dx} = \frac{-1}{x}y + xy^7$

Give an implicit solution

⋮

$$x^{-6}\frac{du}{dx} - 6x^{-7}u = -6x^{-5}$$

Integrate w.r.t x:

$$x^{-6}u = \frac{3}{2}x^{-4} + C_1$$

$y = u^{-1/6}$

$y^{-6} = u$

$$x^{-6}y^{-6} = \frac{3}{2}x^{-4} + C_1$$

Multiply by $2x^6y^6$:

$$2 = 3x^2y^6 + Cx^6y^6$$

Ex: Find an implicit solution:

$$\frac{dy}{dx} = \frac{-2e^x y}{2e^x + y}$$

$$(2e^x + y)dy = -2e^x y\,dx$$

Not Separable
Not Linear/Bernoulli
Not Hom. Degree
Not $u = Ax + By + C$

$$\underbrace{2e^x y\,dx}_{M} + \underbrace{(2e^x + y)}_{N}dy = 0$$

$M_y = 2e^x$ $\quad$ $N_x = 2e^x$ $\quad$ DE is exact

$f = \int 2e^x y\,dx$ $\quad$ AND $\quad$ $f = \int (2e^x + y)\,dy$

$= 2e^x y + g(y)$ $\quad$ $= 2e^x y + \frac{y^2}{2} + h(x)$

$\Rightarrow f = 2e^x y + \frac{y^2}{2}$

DE: $df = 0$

$\int df = \int 0$

$f = c$

$$2e^x y + \frac{y^2}{2} = c$$

Ex: Solve but don't simplify the solution.

$$\frac{dy}{dx} = \frac{x + 3y}{3x + y}$$

$$(3x+y)\,dy = (x+3y)\,dx$$

Not Separable
Not Linear / Bernoulli

Hom. of Degree 1

$$\text{Sub } \begin{cases} y = ux \\ dy = u\,dx + x\,du \end{cases}$$

$$(3x+ux)(u\,dx + x\,du) = (x+3ux)\,dx$$

$$3ux\,dx + 3x^2\,du + u^2x\,dx + ux^2\,du = x\,dx + 3ux\,dx$$

$$3x^2\,du + ux^2\,du = x\,dx - u^2x\,dx$$

$$(3+u)x^2\,du = (1-u^2)x\,dx$$

$$\frac{3+u}{1-u^2}\,du = \frac{dx}{x}$$

Separable

Partial Fractions

$$\frac{3+u}{(1-u)(1+u)} = \frac{A}{1-u} + \frac{B}{1+u}$$

$$3+u = A(1+u) + B(1-u)$$

Sub $u=-1$: $2 = B(2) \Rightarrow B = 1$

Sub $u=1$: $4 = A(2) \Rightarrow A = 2$

$$\int\left(\frac{2}{1-u}+\frac{1}{1+u}\right)du = \int\frac{dx}{x}$$

$$-2\ln|1-u| + \ln|1+u| = \ln|x| + C_1$$

$y = ux$

$\frac{y}{x} = u$

$$-2\ln\left|1-\frac{y}{x}\right| + \ln\left|1+\frac{y}{x}\right| = \ln|x| + C_1$$

Ex: Find an implicit solution

$$\frac{dy}{dx} = \frac{3-x-y}{x+y} \leftarrow 3-(x+y)$$

Not separable
Not linear/Bernoulli
Not hom. degree

Sub $u = Ax+By+C$

Sub $\begin{cases} u = x+y \\ \frac{du}{dx} = 1 + \frac{dy}{dx} \end{cases} \Rightarrow \frac{dy}{dx} = \frac{du}{dx} - 1$

$$\frac{du}{dx} - 1 = \frac{3-u}{u}$$ Separable

$$\frac{du}{dx} - 1 = \frac{3}{u} - 1$$

$$\frac{du}{dx} = \frac{3}{u}$$

$$\int u\,du = \int 3\,dx$$

$$\frac{u^2}{2} = 3x + C_1$$

$$u = x + y$$

$$\frac{(x+y)^2}{2} = 3x + C_1$$

$$(x+y)^2 = 6x + C$$