Asst 1

is on website

Due Thurs Jan 22, 8:30am

Submit pdf or jpeg files to the D2L Dropbox.

Test 1 (4 Questions)

Thurs Jan 29

Covers 1.1–1.2, 2.2–2.5

Bring a calculator, music/earplugs

Practice Problems on website

Formula Sheet will be provided

2.5 Solutions by Substitution

Bernoulli DE

$$\frac{dy}{dx} + Q(x)y = f(x)y^n$$

Like linear DE, but with a factor of y^n on the right side.

When $n \neq 0, 1$

Sub $y = u^{\frac{1}{1-n}}$ to get a linear DE.

Note: When $n=0$ the DE is linear.
When $n=1$ the DE is separable.

Ex: Solve explicitly

$$\frac{1}{y}\frac{dy}{dx} = e^x y + 1$$

$$\frac{dy}{dx} = e^x y^2 + y$$

(*) $$\frac{dy}{dx} - y = e^x y^2$$ Standard Form for a Bernoulli DE

Bernoulli DE $n=2$

Sub $\begin{cases} y = u^{\frac{1}{1-n}} = u^{\frac{1}{-1}} = u^{-1} \\ \frac{dy}{dx} = -u^{-2}\frac{du}{dx} \end{cases}$

Sub into standard form.

$$-u^{-2}\frac{du}{dx} - u^{-1} = e^{x}u^{-2}$$

Multiply by $-u^{2}$:

$$\frac{du}{dx} + u = -e^{x}$$

Linear DE (Standard Form)

$$P(x) = 1$$

$$I.F. = e^{\int P(x)\,dx}$$

$$= e^{\int 1\,dx}$$

$$= e^{x}$$

$$e^{x}\frac{du}{dx} + e^{x}u = -e^{2x}$$

Integrate w.r.t. x:

$$e^{x}u = -\frac{e^{2x}}{2} + C_1$$

$$y = u^{-1}$$

$$y^{-1} = (u^{-1})^{-1}$$

$$y^{-1} = u$$

$$e^{x}y^{-1} = -\frac{e^{2x}}{2} + C_1$$

$$\frac{e^x}{C_1 - \frac{e^{2x}}{2}} = y$$

$$\frac{2e^x}{C - e^{2x}} = y$$

Ex: Solve $\frac{dy}{dx} + y = y^4$.
Do not simplify the solution.

$\frac{dy}{dx} + y = y^4$ Standard form
Bernoulli $n=4$

Sub $\begin{cases} y = u^{\frac{1}{1-n}} = u^{\frac{1}{-3}} = u^{-\frac{1}{3}} \\ \frac{dy}{dx} = -\frac{1}{3}u^{-\frac{4}{3}}\frac{du}{dx} \end{cases}$

into the standard form.

$$-\frac{1}{3}u^{-\frac{4}{3}}\frac{du}{dx} + u^{-\frac{1}{3}} = u^{-\frac{4}{3}}$$ Linear

Multiply by $-3u^{4/3}$:

$$\frac{du}{dx} - 3u = -3$$ (*) Standard Form for Linear DE

$$P(x) = -3$$

$$\text{I.F.} = e^{\int P(x)\,dx}$$

$$= e^{\int -3\,dx}$$

$$= e^{-3x}$$

Multiply the standard form by e^{-3x}:

$$e^{-3x}\frac{du}{dx} - 3e^{-3x}u = -3e^{-3x}$$

Integrate w.r.t. x:

$$e^{-3x}u = e^{-3x} + C_1$$

$$y = u^{-\frac{1}{3}}$$

$$y^{-3} = \left(u^{-\frac{1}{3}}\right)^{-3}$$

$$y^{-3} = u$$

$$e^{-3x}y^{-3} = e^{-3x} + C_1$$

To solve $\frac{dy}{dx} = f(Ax + By + C)$ where A, B, C are real numbers,

Sub $u = Ax + By + C$ to get a separable DE.

Ex: Solve explicitly

$$\frac{dy}{dx} = (x+y+5)^2$$

Sub $u = x+y+5$

$$\frac{du}{dx} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{du}{dx} - 1$$

$$\frac{du}{dx} - 1 = u^2$$ Separable

$$\frac{du}{dx} = u^2 + 1$$

$$\frac{du}{u^2+1} = dx$$

$$\int \frac{du}{u^2+1} = \int dx$$

$$\arctan u = x + C_1$$

$$\arctan(x+y+5) = x + C_1$$

$$x+y+5 = \tan(x+C_1)$$

$$y = \tan(x+C) - x - 5$$

Recap:

DE	Substitution
Homogeneous Degree	$y = ux$ or $x = vy$
Bernoulli $\frac{dy}{dx} + Q(x)y = f(x)y^n$	$y = u^{\frac{1}{1-n}}$
$\frac{dy}{dx} = f(Ax+By+C)$	$u = Ax+By+C$

Note: Don't simplify implicit solutions in this section.

When faced with a DE, check in this order:

(1) Is it separable?

(2)(3) Is it linear / Bernoulli?

(4) Is it homogeneous degree?

(5) Can I sub $u = Ax+By+C$?

(6) Is it exact?

(7) Can I find an I.F. to make it exact?