Asst 1

is on website

Due Thurs Jan 22, 8:30 am

Submit pdf or jpeg files to D2L Dropbox.

Test 1 (4 Questions)

Thurs Jan 29

Covers 1.1–1.2, 2.2–2.5

2.3 Linear DEs Cont'd

Ex: Solve explicitly

$$(x^2-16)\frac{dy}{dx} + xy - x = 0$$

$$\vdots$$

standard form $\quad \frac{dy}{dx} + \frac{x}{x^2-16}\,y = \frac{x}{x^2-16}$

$$\vdots$$

$$\text{I.F.} = \sqrt{x^2-16}$$

$$\sqrt{x^2-16}\,\frac{dy}{dx} + \frac{x}{\sqrt{x^2-16}}\,y = \frac{x}{\sqrt{x^2-16}}$$

Integrate w.r.t. x

Shortcut

$$\sqrt{x^2-16}\; y = \sqrt{x^2-16} + C$$

Sub $u = x^2-16$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$$\int \frac{x}{\sqrt{x^2-16}}\,dx$$

$$= \frac{1}{2}\int \frac{du}{\sqrt{u}}$$

$$= \sqrt{u} + C$$

$$y = 1 + \frac{C}{\sqrt{x^2-16}}$$

Terminology:

$\frac{C}{\sqrt{x^2-16}}$ is a <u>transient term</u> :

$$\frac{C}{\sqrt{x^2-16}} \to 0 \quad \text{as} \quad x \to \infty$$

So $y \to 1$ as $x \to \infty$.

<u>Ex</u>: Solve explicitly:

$$t\frac{dy}{dt} + 3y = t+1, \quad y(-1) = \frac{1}{3}$$

$$\frac{dy}{dt} + \frac{3}{t}y = 1 + \frac{1}{t}$$ Standard Form

$$P(t) = \frac{3}{t}$$

$$\text{I.F.} = e^{\int \frac{3}{t}dt}$$

$$= e^{3\ln|t|}$$

$$= e^{\ln|t|^3}$$

$$= |t|^3$$

$$= \begin{cases} t^3, & t > 0 \\ -t^3, & t < 0 \end{cases}$$

$$= -t^3$$

$\boxed{\text{Interval of Solution: } t < 0}$

(See initial condition)

$$-t^3\frac{dy}{dt} - 3t^2 y = -t^3 - t^2$$

Integrate w.r.t. t:

$$-t^3 y = -\frac{t^4}{4} - \frac{t^3}{3} + C$$

$$\boxed{y = \frac{t}{4} + \frac{1}{3} - \frac{C}{t^3}}$$

Sub $y = \frac{1}{3}$, $t = -1$: $\frac{1}{3} = -\frac{1}{4} + \frac{1}{3} - \frac{C}{(-1)}$

$$\frac{1}{4} = C$$

$$y = \frac{t}{4} + \frac{1}{3} - \frac{1}{4t^3}$$

<u>Ex</u>: TRICKY

Solve for x.

$$y\,dx - (x + y^6)\,dy = 0$$

$$y - (x + y^6)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} - \underbrace{\frac{1}{x + y^6}}_{!}\, y = 0$$

Not linear in y ☹

Linear in x ?

$$\frac{dx}{dy} + P(y)\,x = f(y) \quad ?$$

$$y\,dx - (x+y^6)\,dy = 0$$

$$y\frac{dx}{dy} - (x+y^6) = 0$$

$$\frac{dx}{dy} - \frac{x+y^6}{y} = 0$$

$$\frac{dx}{dy} - \frac{x}{y} - y^5 = 0$$

$$\frac{dx}{dy} - \frac{1}{y}x = y^5 \qquad (*)$$

$$\boxed{\begin{array}{c}\text{standard Form} \\ \frac{dx}{dy} + P(y)x = f(y)\end{array}}$$

$$P(y) = -\frac{1}{y}$$

$$I.F. = e^{\int -\frac{1}{y}\,dy}$$

$$= e^{-\ln|y|}$$

$$= e^{\ln |y|^{-1}}$$

$$= |y|^{-1}$$

$$= \begin{cases} \frac{1}{y}, & y > 0 \\ \frac{-1}{y}, & y < 0 \end{cases}$$

$$= \frac{1}{y}$$

$\boxed{\text{Interval of Solution: } y > 0}$

$$\frac{1}{y}\frac{dx}{dy} - \frac{1}{y^2}x = y^4$$

Integrate w.r.t. y:

$$\frac{1}{y}x = \frac{y^5}{5} + C$$

$$x = \frac{y^6}{5} + Cy$$

2.4 Exact DEs

Ex: $f(x,y) = 2x^3 + 6xy^2 - 9y^4$

Partial derivative of f w.r.t. x:

$$f_x \text{ or } \frac{\partial f}{\partial x} = 6x^2 + 6y^2$$

(treat y as a constant)

$$f_y \text{ or } \frac{\partial f}{\partial y} = 12xy - 36y^3$$

(treat x as a constant)

Ex: $f(x,y) = x\sin 2y + \tan 8x + y^2$

Find f_x and f_y.

$$f_x = \sin 2y + 8\sec^2 8x$$

$$f_y = 2x\cos 2y + 2y$$

The differential of f is

$$df = f_x\,dx + f_y\,dy$$

Ex: Let $f = x^3 y + \sin(3x+y)$
Find df.

$$df = f_x\,dx + f_y\,dy$$

$$df = \left[3x^2 y + 3\cos(3x+y)\right]dx + \left[x^3 + \cos(3x+y)\right]dy$$

A DE $\quad M\,dx + N\,dy = 0$
is <u>exact</u> if the left side
is a differential.

Quick Way to Check:
A DE is exact if $M_y = N_x$.

Ex: $(3x^2 + \sin y)\,dx + x\cos y\,dy = 0$
Is the DE exact?

$M = 3x^2 + \sin y \qquad N = x\cos y$

$M_y = \cos y \qquad N_x = \cos y$

$M_y = N_x$ ✓
YES

<u>Ex</u>: $x^3 y\,dx + (x+1)\,dy = 0$

Is the DE exact?

$M = x^3 y \qquad N = x+1$

$M_y = x^3 \qquad N_x = 1$

$M_y \neq N_x$

No