Assignment 4

Will be due Tues April 7, 8:30am

Final Exam

Tues April 14
8:30 - 11:30am
CHW 233

Final Exam: 10 Questions
3 Hours

Chapter	# of Questions
2	2
3	1
4	2
5	1
6	1
7	2
8	1

Leah will be available by email on Monday April 13

8.2 Solving Homogeneous Systems

Case 4: $\lambda = \alpha \pm \beta i$

Ex: Solve $\vec{X}' = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 8 \\ 0 & -1 & -1 \end{bmatrix} \vec{X}$

⋮

$\lambda = 2$: $\vec{X}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{2t}$

$\lambda = 1+2i$: $\vec{K} = \begin{bmatrix} 0 \\ 2+2i \\ -1 \end{bmatrix}$

$(\alpha = 1, \beta = 2)$

$$\vec{K} = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} + i \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$\vec{B}_1 = Re(\vec{K})$ $\quad$ $\vec{B}_2 = Im(\vec{K})$ (a real vector)

$$\vec{X}_2 = e^{\alpha t}(\vec{B}_1 \cos\beta t - \vec{B}_2 \sin\beta t) = e^{t}\left(\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}\cos 2t - \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}\sin 2t\right)$$

$$\vec{X}_3 = e^{\alpha t}(\vec{B}_1 \sin\beta t + \vec{B}_2 \cos\beta t) = e^{t}\left(\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}\sin 2t + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}\cos 2t\right)$$

$$\vec{X} = C_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{2t} + C_2 \begin{bmatrix} 0 \\ 2\cos 2t - 2\sin 2t \\ -\cos 2t \end{bmatrix} e^{t} + C_3 \begin{bmatrix} 0 \\ 2\sin 2t + 2\cos 2t \\ -\sin 2t \end{bmatrix} e^{t}$$

Exam Review

Methods

Separable?
Linear / Bernoulli?
Homogeneous Degree?
Sub $u = Ax + By + C$?
Exact?
Find an I.F. to make it exact?

① $(y+y\cos xy)dx+(x+x\cos xy+\frac{2}{y})dy=0$

$M_y = 1+\cos xy - xy\sin xy$

$N_x = 1+\cos xy - xy\sin xy$

$M_y = N_x \Rightarrow$ DE is exact

$f=\int (y+y\cos xy)dx$ and $f=\int (x+x\cos xy+\frac{2}{y})dy$

$= xy + \sin xy + g(y)$ $= xy+\sin xy+2\ln|y|+h(x)$

$\Rightarrow f = xy+\sin xy+2\ln|y|$

DE : $df=0$

Integrate : $f=c$

$$\boxed{xy+\sin xy+2\ln|y| = c}$$

② Linear $\frac{dy}{dx}+P(x)y = f(x)$

Bernoulli $\frac{dy}{dx}+P(x)y = f(x)y^n$

Bernoulli $n=3$

Sub $\begin{cases} y = u^{\frac{1}{1-n}} = u^{\frac{1}{1-3}} = u^{-\frac{1}{2}} \end{cases}$

$$\frac{dy}{dx} = -\frac{1}{2}u^{-\frac{3}{2}}\frac{du}{dx}$$

$$y' + \frac{y}{x} = x^3 y^3$$

$$-\frac{1}{2}u^{-\frac{3}{2}}\frac{du}{dx} + \frac{1}{x}u^{-\frac{1}{2}} = x^3 u^{-\frac{3}{2}}$$

Multiply by $-2u^{\frac{3}{2}}$:

$$\boxed{\frac{du}{dx} - \frac{2}{x}u = -2x^3}$$

Linear (standard Form)

$$P(x) = -\frac{2}{x}$$

$$\text{I.F.} = e^{\int P(x)\,dx}$$

$$= e^{\int -\frac{2}{x}\,dx}$$

$$= e^{-2\ln|x|}$$

$$= e^{\ln|x|^{-2}}$$

$$= x^{-2}$$

$$x^{-2}\frac{du}{dx} - 2x^{-3}u = -2x$$

Integrate:

$$x^{-2}u = -x^2 + C$$

$y = u^{-\frac{1}{2}}$

$y^{-2} = u$

$$x^{-2} y^{-2} = -x^2 + C$$

(3) $(x^2+y^2)\,dx - xy\,dy = 0$

Homogeneous of degree 2

Sub $\begin{cases} y = ux \\ dy = u\,dx + x\,du \end{cases}$ $\quad \frac{dy}{dx} = u + x\frac{du}{dx}$

$$(x^2 + u^2x^2)\,dx - \cancel{x(ux)}_{ux^2}(u\,dx + x\,du) = 0$$

$$x^2\,dx + u^2x^2\,dx - u^2x^2\,dx - ux^3\,du = 0$$

$$x^2\,dx = ux^3\,du$$

$$\frac{dx}{x} = u\,du \qquad \text{Separable}$$

Integrate: $\ln|x| = \frac{u^2}{2} + C_1$

$y = ux$

$u = \frac{y}{x}$

$$\ln|x| = \frac{1}{2}\frac{y^2}{x^2} + C_1$$

$$\ln|x| + C_2 = \frac{1}{2}\frac{y^2}{x^2}$$

$$2x^2 \ln|x| + C_3 x^2 = y^2$$

$$y = \pm\sqrt{2x^2 \ln|x| + Cx^2}$$

Note: This DE could also be written as a Bernoulli DE with $n = -1$:

$$\frac{dy}{dx} - \frac{y}{x} = \frac{x}{y}$$

and you will get the same solution as above.