Test 3 (4 Questions)

FRI MARCH 20

Covers 4.6, 4.7, 5.1, 6.2

Practice Problems on website

Assignment 4

Will be due Tues April 7

Final Exam

Tues April 14

8:30-11:30 am

CHW 233

7.3 Translations (Shifts)

Ex: Solve using the Laplace transform

$$y'' + 4y' + 20y = 50e^{t}, \quad y(0)=9, \; y'(0)=-28$$

⋮

5) Find $y(t)$

From ⍟ $\quad y(t) = \mathcal{L}^{-1}\left\{ \frac{2}{s-1} + \frac{7s-2}{s^2+4s+20} \right\}$

$$s^2+4s+2 = (s+2)^2 + ?$$
$$= (s+2)^2 + 16$$
$$= (s+2)^2 + 4^2$$

$$7s-2 = ?(s+2) + ?$$
$$= 7(s+2) + ?$$
$$= 7(s+2) - 16$$

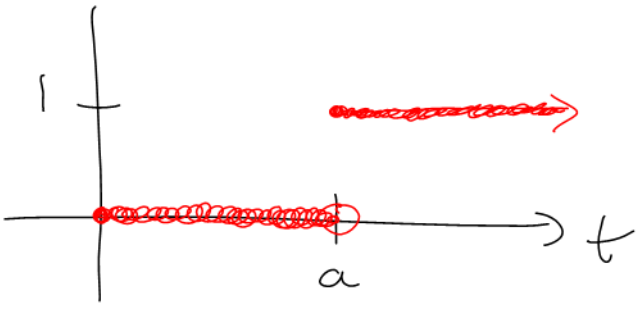
$$y(t) = \mathcal{L}^{-1}\left\{ \frac{2}{s-1} + \frac{7(s+2)-16}{(s+2)^2+4^2} \right\}$$

$$= \mathcal{L}^{-1}\left\{ \frac{2}{s-1} + 7\cdot\frac{s+2}{(s+2)^2+4^2} - 4\cdot\frac{4}{(s+2)^2+4^2} \right\}$$

$$= 2e^{t} + 7e^{-2t}\cos 4t - 4e^{-2t}\sin 4t$$

Heaviside Function

$$\mathcal{U}(t-a) = \begin{cases} 0, & 0 \le t < a \\ 1, & t \ge a \end{cases}$$



Also called the "unit step function".

FACT

A step function can be written as:

$$f(t) = (\text{Initial Function}) + (\text{Change at } t=t_1)\,u(t-t_1) + (\text{Change at } t=t_2)\,u(t-t_2) + \dots$$

Ex: Write $f(t)$ in terms of $u(t-a)$

a) $f(t) = \begin{cases} 0, & 0 \le t < 1 \\ t^2, & t \ge 1 \end{cases}$ "step function"

$f(t) = 0 + t^2 u(t-1)$

$f(t) = t^2 u(t-1)$

b) $f(t) = \begin{cases} t^2, & 0 \le t < 2 \\ 0, & t \ge 2 \end{cases}$

$f(t) = ? + ?\,u(t-2)$

$f(t) = t^2 - t^2 u(t-2)$

c) $f(t) = \begin{cases} 1, & 0 \le t < 2 \\ 3, & 2 \le t < 5 \\ 0, & t \ge 5 \end{cases}$

$f(t) = ? + ?\,u(t-2) + ?\,u(t-5)$

$$f(t) = 1 + 2u(t-1) - 3u(t-5)$$

$$\mathcal{L}\{f(t)u(t-a)\} = e^{-as}\mathcal{L}\{f(t+a)\}$$

$$\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a)$$

Ex: a) $\mathcal{L}\{u(t-6)\}$

$$= e^{-6s}\mathcal{L}\{f(t+6)\}$$

$$= e^{-6s}\mathcal{L}\{1\}$$

$$= \frac{e^{-6s}}{s}$$

$\begin{cases} f(t) = 1 \\ f(t+6) = 1 \end{cases}$

b) $\mathcal{L}\{t^2 u(t-1)\}$

$$= e^{-s}\mathcal{L}\{f(t+1)\}$$

$\begin{cases} f(t) = t^2 \\ f(t+1) = (t+1)^2 \\ \quad = t^2+2t+1 \end{cases}$

$$= e^{-s}\mathcal{L}\{t^2+2t+1\}$$

$$= \left(\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right)e^{-s}$$

c) $\mathcal{L}^{-1}\left\{\frac{1}{s-2}e^{-3s}\right\}$

$$= f(t-3)u(t-3)$$

$$\begin{cases} F(s) = \frac{1}{s-2} \\ f(t) = e^{2t} \\ f(t-3) = e^{2(t-3)} \end{cases}$$

$$= e^{2(t-3)}\,u(t-3)$$

Ex: Solve using the Laplace transform

$$y' + 5y = f(t), \quad y(0) = 0$$

where $f(t) = \begin{cases} 0, & 0 \le t < 3 \\ 2, & t \ge 3 \end{cases}$

$$f(t) = ? + ?\,u(t-3)$$

$$f(t) = 0 + 2\,u(t-3)$$

$$f(t) = 2\,u(t-3)$$

Dt: $y' + 5y = 2\,u(t-3)$

1) Apply $\mathcal{L}$

$$s\,Y(s) - y(0) + 5Y(s) = \mathcal{L}\{2\,u(t-3)\}$$

$$\mathcal{L}\{2u(t-3)\} = e^{-3s}\,\mathcal{L}\{f(t+3)\}$$

$$= e^{-3s}\,\mathcal{L}\{2\}$$

$$= \frac{2e^{-3s}}{s}$$

$$\begin{cases} f(t) = 2 \\ f(t+3) = 2 \end{cases}$$

$$s\,Y(s) - y(0) + 5Y(s) = \frac{2e^{-3s}}{s}$$

$y(0)=0$: $$s\,Y(s) + 5Y(s) = \frac{2e^{-3s}}{s}$$

2) Solve for $Y(s)$

$$(s+5)\,Y(s) = \frac{2e^{-3s}}{s}$$

$$Y(s) = \frac{2e^{-3s}}{s(s+5)}$$