Math 254 Test Two   Total Marks: 25

Name:______________________________

Marks may be deducted for not showing all your work.

1. [3 marks] The lifetime of a machine part is exponentially distributed with probability density function $f(x) = 0.2e^{-0.2x}$, where x is measured in years. Find the probability that the part lasts at most 8 years. Round your answer to three decimal places.

$$P(X \le 8) = \int_0^8 0.2e^{-0.2x}\,dx$$
$$= \left[-e^{-0.2x}\right]_0^8$$
$$= -e^{-1.6} + 1$$
$$\approx 0.798$$

2. [3 marks] A website receives an average of 120 visits per day. Find the probability that the website receives at least three visits in the next hour. Round your answer to three decimal places.

Poisson

$\mu = 120$ visits/day $= 5$ visits/hour

$\boxed{\text{Use } \mu = 5}$ Let $X =$ #visits

$$P(X \ge 3) = 1 - P(X=0) - P(X=1) - P(X=2)$$
$$= 1 - \frac{5^0 e^{-5}}{0!} - \frac{5^1 e^{-5}}{1!} - \frac{5^2 e^{-5}}{2!}$$
$$= 1 - e^{-5}\left(\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!}\right)$$
$$\approx 0.875$$

3. [3 marks] At an engineering firm, employees worked a mean of 57 hours last week, with a standard deviation of 4 hours. Find the probability that a [redacted] of 40 employees worked a total of more than 2300 hours last week.

group randomly selected

$\mu = 57 \quad \sigma = 4 \quad n = 40$

$P(\text{total} > 2300)$

total = 2300 is equivalent to average = $\frac{2300}{40} = 57.5$

$= P(\bar{x} > 57.5)$

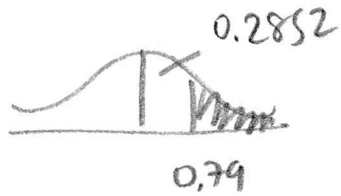
By Central Limit Theorem

$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \quad (n \geq 30 \checkmark)$

$= \frac{57.5 - 57}{(4/\sqrt{40})}$

≈ 0.79

$= P(z > 0.79)$

$=$ 

$= 0.5 - 0.2852$

$= 0.2148$

4. [4 marks] A coin lands on heads 52% of the time. Estimate the probability of observing between 180 and 195 heads (inclusive) in 400 tosses of the coin.

$X = \#$ heads

Binomial $n = 400$, $p = 0.52$ $q = 1 - p = 0.48$

$P(180 \leq X \leq 195)$

$np > 5$, $nq > 5$ ✓

$\mu = np = 208$

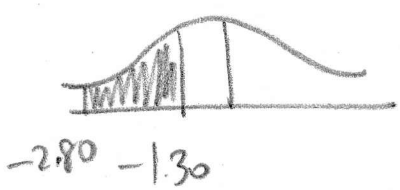
$\sigma = \sqrt{npq} \approx 9.9920$

$z_1 = \dfrac{195 - \mu}{\sigma} \approx -1.30$

$z_2 = \dfrac{180 - \mu}{\sigma} \approx -2.80$

$= P(-2.80 \leq z \leq -1.30)$

$=$



$= 0.4974 - 0.4032$

$= 0.0942$

5. [4 marks] We want to estimate a population proportion with a 99% margin of error at most 0.05. We have no information about $\hat{p}$. What is the minimum sample size we can use when collecting the sample data?

$$z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \le 0.05$$

No info about $\hat{p}$:
Use $\hat{p} = 0.5 = \hat{q}$

$$\frac{2.326\sqrt{(0.5\times0.5)}}{\sqrt{n}} \le 0.05$$

$1-\alpha = 0.98$
$z_{\alpha/2} = 2.326$

$$\frac{2.326\sqrt{(0.5\times0.5)}}{0.05} \le \sqrt{n}$$

$$\left(\frac{2.326\sqrt{(0.5\times0.5)}}{0.05}\right)^2 \le n$$

$n \ge 541.0276$

$\boxed{\text{542 is the minimum}}$

6. [2 marks] We are testing the hypothesis H_0: $\mu = 220$ with $\alpha = 0.05$. We are given $\overline{x} = 212$, $s = 16$ and $n = 64$.

a) Define a Type I error.

Reject H_0 when it is in fact true.

b) Find the ~~[illegible]~~ Probability of a Type I error.

$P(\text{Type I error}) = \alpha = 0.05$

7. [6 marks] Test whether the two population means are equal at the 2% significance level given the following sample data:
$n_1 = 100, \overline{x}_1 = 81.4, s_1 = 3.1, n_2 = 60, \overline{x}_2 = 82.2, s_2 = 1.9.$

a) State H_0 and H_a

$H_0: \mu_1 = \mu_2 \qquad H_a: \mu_1 \neq \mu_2$ (two-tailed test)

b) Check any necessary assumptions.

$n_1 \geq 30, \quad n_2 \geq 30$

c) Do you reject H_0? Show all your work.

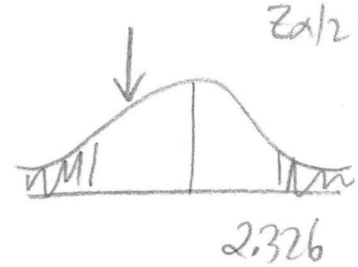
$D_0 = 0$ from $H_0: \mu_1 - \mu_2 = \boxed{0}$

$$z = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}}$$

$$= \frac{81.4 - 82.2 - 0}{\sqrt{\left(\frac{3.1^2}{100} + \frac{1.9^2}{60}\right)}}$$

≈ -2.024

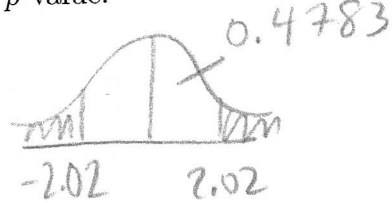
$1 - \alpha = 0.98$
$z_{\alpha/2} = 2.326$



Don't reject H_0
$\mu_1 = \mu_2$

d) Find the p-value.

$p =$



$= 1 - 2(0.4783)$
$= 0.0434$