Solutions

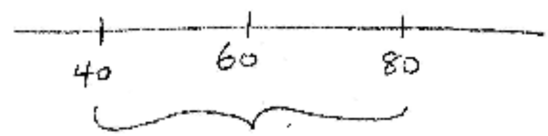
(1) a) $45 = 60 - 3(5) = \mu - 3\sigma$

$75 = 60 + 3(5) = \mu + 3\sigma$

$\geq 1 - \frac{1}{3^2}$ of measurements lie in the interval $\mu \pm 3\sigma$

$\geq \frac{8}{9}$ of measurements lie in the interval

b) $80 = 60 + 4(5) = \mu + 4\sigma$



$\geq 1 - \frac{1}{4^2}$ of measurements lie in the interval $40 \leq X \leq 80$

$\geq \frac{15}{16}$ of measurements lie in $40 \leq X \leq 80$

$\leq \frac{1}{16}$ of measurements are larger than 80.

(2) a) $40 = 47 - \frac{7}{4}(4) = \mu - \frac{7}{4}\sigma$

$54 = 47 + \frac{7}{4}(4) = \mu + \frac{7}{4}\sigma$

$\geq 1 - \frac{1}{\left(\frac{7}{4}\right)^2}$ of measurements lie in the interval

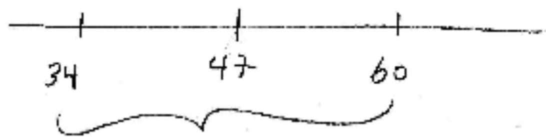
$\geq \left(1 - \frac{1}{\left(\frac{7}{4}\right)^2}\right)(300)$ measurements lie in the interval

≥ 202.04 "

≥ 203 measurements lie in the interval

b) $30 = 47 - \frac{17}{4}(4) = \mu - \frac{17}{4}\sigma$

$60 = 47 + \frac{13}{4}(4) = \mu + \frac{13}{4}\sigma$

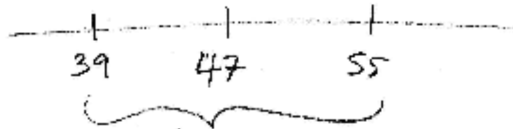


$\geq \left(1 - \frac{1}{\left(\frac{13}{4}\right)^2}\right)(300)$ measurements lie in $34 \leq X \leq 60$

≥ 272 measurements lie in $34 \leq X \leq 60$

≥ 272 measurements lie in $30 \leq X \leq 60$

c) $55 = 47 + 2(4) = \mu + 2\sigma$



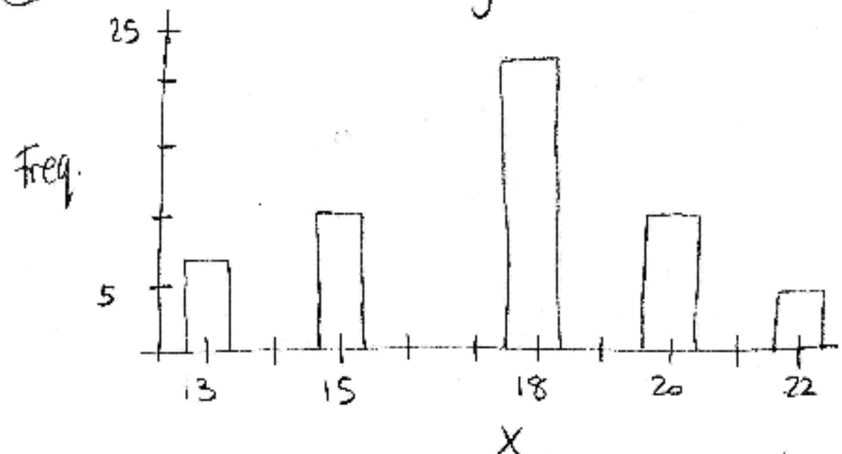
$\geq \left(1 - \frac{1}{2^2}\right)(300)$ measurements lie in $39 \leq X \leq 55$

≥ 225 "

≤ 75 measurements are bigger than 55

(3) No assumptions are needed.
Tchebysheff's Theorem applies to all data sets.

(4) a) Draw a histogram.



Data is roughly mound-shaped.
Yes, the Empirical Rule applies.

b) Approximately 68% of data lie within $\mu \pm \sigma$

c) From calculator: $\mu \approx 17.66$ $\sigma \approx 2.64$

$\mu + \sigma \simeq 20.3$

$\mu - \sigma \simeq 15.02$

measurements in $15.02 \le X \le 20.3$ is $23 + 12 = 35$

The actual proportion is $\frac{35}{59} \approx 59\%$

(5) a) $\simeq 95\%$ of measurements lie in $\mu \pm 2\sigma$

b) $\simeq 99.7\%$ of measurements lie in $\mu \pm 3\sigma$

(6) $0.75 = 1 - \frac{1}{k^2}$

$\frac{1}{k^2} = 0.25$

$k^2 = 4$

$k = \pm 2$

$\geq 75\%$ of measurements lie in $\mu \pm 2\sigma$, with $\mu = 10$, $\sigma = 2$:

$\geq 75\%$ of measurements lie in the interval $6 \le X \le 14$.

(7) $0.90 = 1 - \frac{1}{k^2}$

$\frac{1}{k^2} = 0.1$

$k^2 = 10$

$k = \pm\sqrt{10}$

$\mu \pm \sqrt{10}\sigma$ is the smallest interval.

(8) $0.50 = 1 - \frac{1}{k^2}$

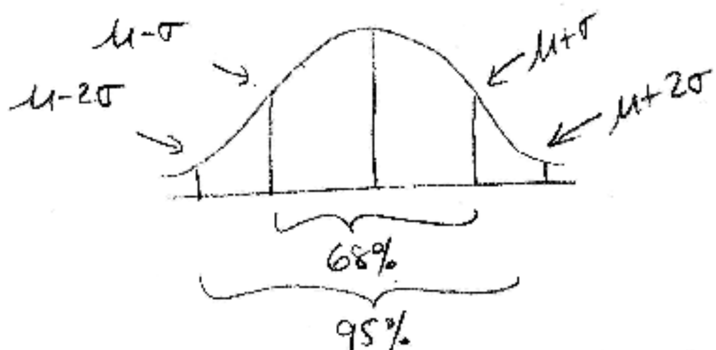
$\frac{1}{k^2} = 0.5$

$k^2 = 2$

$k = \pm\sqrt{2}$

$\mu \pm \sqrt{2}\sigma$ is the smallest interval.

(9) A mound-shaped distribution should look like this:



Some measurements would be less than $\mu - \sigma$. But in this example $\mu - \sigma = -1$. Rats cannot take -1 days to recover. So no, data is not mound-shaped.

(10) $0.75 = 1 - \frac{1}{k^2}$

$\frac{1}{k^2} = 0.25$

$k^2 = 4$

$k = \pm 2$

$\mu \pm 2\sigma$ will contain $\geq 75\%$ of recovery times.

With $\mu = 3$ $\sigma = 4$:

$\boxed{-5 \leq X \leq 11 \text{ will contain } \geq 75\% \text{ of recovery times.}}$

<u>Note</u>: Since there are no negative recovery times we know that

$\boxed{0 \leq X \leq 11 \text{ will contain } \geq 75\% \text{ of recovery times.}}$

Both answers are acceptable, although the second one is more precise.