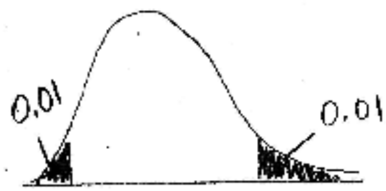
Solutions

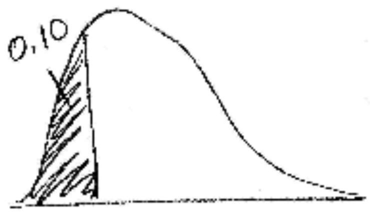
(1)



$\chi^2_{0.99} = 0.872$ $\chi^2_{0.01} = 16.812$

two-tailed
df = 6

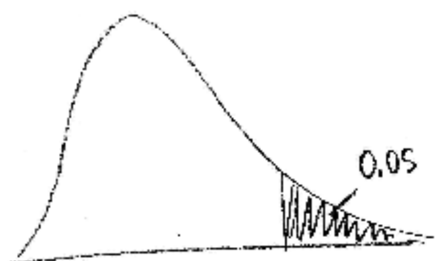
(2)



$\chi^2_{0.90} = 3.490$

left-tailed
df = 8

(3)



$\chi^2_{0.05} = 18.307$

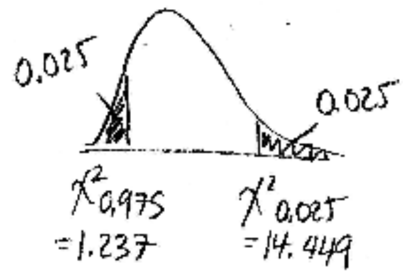
right-tailed
df = 10

(4) $n = 7 \quad s^2 = 4.2$ (normal population)

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$$\sigma^2 = \frac{(n-1)s^2}{\chi^2}$$

$\alpha = 0.05$
$df = n-1 = 6$



$\chi^2 = 14.449$: $\quad \sigma^2 = \frac{6(4.2)}{14.449} \approx 1.7$

$\chi^2 = 1.237$ $\quad \sigma^2 = \frac{6(4.2)}{1.237} \approx 20.4$

$$\boxed{1.7 \le \sigma^2 \le 20.4}$$

(5) We need the population to be normal

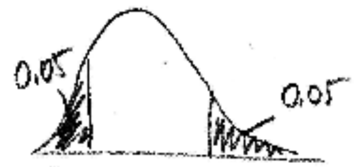
(6) $n = 9$ $\quad s = 7.201$ $\quad$ (normal population)

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$\alpha = 0.1$

$df = n - 1 = 8$

$$\sigma^2 = \frac{(n-1)s^2}{\chi^2}$$



$\chi^2_{0.95} = 2.733$ $\quad$ $\chi^2_{0.05} = 15.507$

$\chi^2 = 15.507$: $\quad \sigma^2 = \dfrac{8(7.201)^2}{15.507} \simeq 26.751$

$\chi^2 = 2.733$: $\quad \sigma^2 = \dfrac{8(7.201)^2}{2.733} \simeq 151.787$

$26.751 \leq \sigma^2 \leq 151.787$

Taking square roots:

$$\boxed{5.172 \leq \sigma \leq 12.320}$$

⑦ $n=16 \quad s=12.355$ (normal population)

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$\alpha = 0.05$
$df = n-1 = 15$

$$\sigma^2 = \frac{(n-1)s^2}{\chi^2}$$

$$\sigma^2 = \frac{15(12.355)^2}{7.261}$$

$\chi^2_{0.95}$
$= 7.261$

≈ 315.341

$$\boxed{\sigma^2 \le 315.341}$$

⊛ Note: Use the <u>left</u> tail because a smaller χ^2 gives a larger σ^2

⑧ $n=12 \quad s^2=1.46$ (normal population)

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$\alpha = 0.1$
$df = n-1 = 11$

$$\sigma^2 = \frac{(n-1)s^2}{\chi^2}$$

$$\sigma^2 = \frac{11(1.46)}{17.275}$$

$\chi^2_{0.1}$
$= 17.275$

≈ 0.930

$$\boxed{0.930 \le \sigma^2}$$

⊛ Note: Use the <u>right</u> tail because a larger χ^2 gives a smaller σ^2

(9) 1) $H_0: \sigma^2 = 100$ $\quad$ $H_a: \sigma^2 \neq 100$

two-tailed

2) Assumptions: normal population ✓

3) $\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$ $\quad$ From calculator: $s^2 = 278.3$ $\quad$ $n = 5$

$= \dfrac{4(278.3)}{100}$

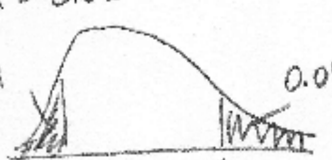
$= 11.132$

4) $df = n - 1 = 4$

$\alpha = 0.02$

two-tailed

0.01 $\quad$ 0.01



$\chi^2_{0.99} = 0.297$ $\quad$ $\chi^2_{0.01} = 13.277$

5) Don't reject H_0.

$\sigma^2 = 100$.

6) p-value:

$df = 4$

$\chi^2_{0.05}$	$\chi^2_{0.025}$
9.488	11.143

11.132

$0.025 < \frac{p}{2} < 0.05$

$0.05 < p < 0.1$ ← double the area for a two-tailed test

(10) 1) $H_0: \sigma^2 = 6$ $H_a: \sigma^2 < 6$

left-tailed

2) Assumptions: normal population ✓

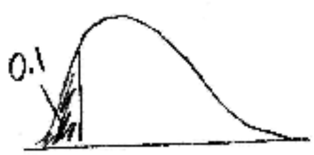
3) $\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$ $n = 5$ $s^2 = 2.12$

$= \dfrac{4(2.12)}{6}$

≈ 1.413

4) $df = n-1 = 4$

$\alpha = 0.1$



left-tailed

$\chi^2_{0.9}$
$= 1.064$

5) Don't reject H_0.

$\sigma^2 = 6$

6) p-value:

$\chi^2_{0.9}$ $\chi^2_{0.1}$

df = 4 $\boxed{1.064 \uparrow 7.779}$

1.413

$\boxed{0.1 < p < 0.9}$