

Solutions

① Paired by Location

$$\bar{d} = 1.58 \quad s_d \approx 0.817$$

$$df = n - 1 = 4$$

$$\alpha = 0.05$$

$$t_{\alpha/2} = 2.776$$

$$\bar{d} \pm t_{\alpha/2} \frac{s_d}{\sqrt{n}}$$

$$1.58 \pm 2.776 \left(\frac{0.817}{\sqrt{5}} \right)$$

$$\boxed{0.6 \leq \mu_1 - \mu_2 \leq 2.6}$$

②

$$n < 30 \checkmark$$

Differences must be normally distributed

* PAIRED BY LOCATION *

③ 1) $H_0: \mu_1 - \mu_2 = 0$ $H_a: \mu_1 - \mu_2 > 0$
right-tailed

2) Assumption:
 $n < 30$ ✓
differences are normally distributed ✓

3) $n = 5$ $\bar{d} = 1.58$ $s_d \approx 0.817$

$$t = \frac{\bar{d} - D_0}{s_d/\sqrt{n}}$$

D_0 comes from H_0 :
 $\mu_1 - \mu_2 = 0 \leftarrow D_0$

$$= \frac{1.58 - 0}{(0.817/\sqrt{5})}$$
$$\approx 4.324$$

4) $df = 4$
 $\alpha = 0.05$

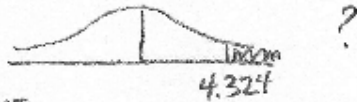
right-tailed

$t_{\alpha} = 2.132$



5) Reject H_0 .
 $\mu_1 - \mu_2 > 0$ or $\mu_1 > \mu_2$

6) p-value?



$df = 4$

$t_{0.01}$	$t_{0.005}$
3.747	4.604

4.324

$0.005 < p < 0.01$

④ * INDEPENDENT SAMPLES *

1) $H_0: \mu_1 - \mu_2 = 0$ $H_a: \mu_1 - \mu_2 \neq 0$

two-tailed

2) Assumptions: $n_1 < 30$ or $n_2 < 30$ ✓
 both populations normal ✓
 $\frac{\text{larger } s^2}{\text{smaller } s^2} \leq 3$ ✓

3)
$$t = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

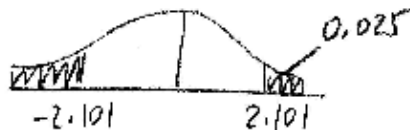
$$= \frac{59.646 - 59.627 - 0}{\sqrt{0.031 \left(\frac{1}{10} + \frac{1}{10} \right)}}$$

$$\approx 0.241$$

$$s^2 = \frac{9s_1^2 + 9s_2^2}{18} = 0.031$$

Do ones from H_0 :
 $\mu_1 - \mu_2 = 0 \leftarrow D_0$

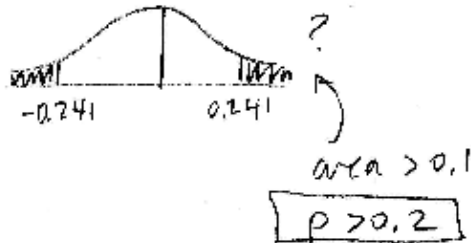
4) two-tailed
 $df = n_1 + n_2 - 2 = 18$
 $\alpha = 0.05$
 $t_{\alpha/2} = 2.101$



5) Don't reject H_0 .
 $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

6) p-value?

$df = 18$ $t_{0.1}$
 \uparrow 1.330
 0.241



⑤

* INDEPENDENT SAMPLES *

$$df = n_1 + n_2 - 2 = 18$$

$$\alpha = 0.01$$

$$t_\alpha = 2.552$$

$$s^2 = \frac{9(0.027) + 9(0.035)}{18} \\ = 0.031$$

99% UCB for $\mu_1 - \mu_2$:

$$\bar{x}_1 - \bar{x}_2 + t_\alpha \sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$$

$$59.646 - 59.627 + 2.552 \sqrt{(0.031 \left(\frac{1}{10} + \frac{1}{10} \right))}$$

$$\boxed{\mu_1 - \mu_2 \leq 0.220}$$

⑥

* INDEPENDENT SAMPLES *

$$df = n_1 + n_2 - 2 = 18$$

$$\alpha = 0.05$$

$$t_\alpha = 1.734$$

$$s^2 = \frac{9(0.027) + 9(0.035)}{18} \\ = 0.031$$

95% LCB for $\mu_1 - \mu_2$:

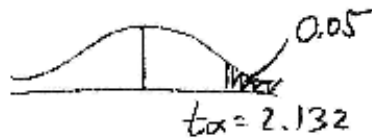
$$\bar{x}_1 - \bar{x}_2 - t_\alpha \sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$$

$$59.646 - 59.627 - 1.734 \sqrt{(0.031 \left(\frac{1}{10} + \frac{1}{10} \right))}$$

$$\boxed{-0.118 \leq \mu_1 - \mu_2}$$

- ⑦ $n_1 < 30$ or $n_2 < 30$
 both populations normal ✓
 $\frac{\text{larger } s^2}{\text{smaller } s^2} \leq 3$ ✓

- ⑧ * PAIRED BY WEEK *
 $n=5$ $\bar{d}=0.008$ $Sd \approx 0.045$
 $df = n-1 = 4$
 $\alpha = 0.05$
 $t_{\alpha} = 2.132$

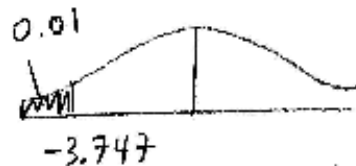


$$\bar{d} + t_{\alpha} \frac{Sd}{\sqrt{n}}$$

$$0.008 + 2.132 \left(\frac{0.045}{\sqrt{5}} \right)$$

$$\boxed{\mu_1 - \mu_2 \leq 0.05}$$

- ⑨ * PAIRED BY WEEK *
 $n=5$ $\bar{d}=0.008$ $Sd \approx 0.045$
 $df = n-1 = 4$
 $\alpha = 0.01$
 $t_{\alpha} = 3.747$



$$\bar{d} - t_{\alpha} \frac{Sd}{\sqrt{n}}$$

$$0.008 - 3.747 \left(\frac{0.045}{\sqrt{5}} \right)$$

$$\boxed{-0.07 \leq \mu_1 - \mu_2}$$

⑩ * INDEPENDENT SAMPLES *

$$n_1 = 4 \quad \bar{x}_1 = 1.3075$$

$$n_2 = 4 \quad \bar{x}_2 = 1.27$$

$$s_1^2 = 0.006$$

$$s_2^2 = 0.0022$$

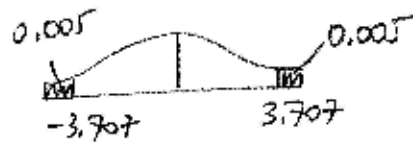
(Assumptions: $n_1 < 30$ or $n_2 < 30$ ✓
both populations normal ✓

$\frac{\text{larger } s^2}{\text{smaller } s^2} \leq 3$ ✓)

$$df = n_1 + n_2 - 2 = 6$$

$$\alpha = 0.01$$

$$t_{\alpha/2} = 3.707$$



$$\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2} \sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} \quad s^2 = \frac{3(0.006) + 3(0.0022)}{6}$$
$$= 0.0041$$

$$1.3075 - 1.27 \pm 3.707 \sqrt{0.0041 \left(\frac{1}{4} + \frac{1}{4} \right)}$$

$$\boxed{-0.13 \leq \mu_1 - \mu_2 \leq 0.21}$$