

Solutions

① $n=80$ $\bar{x}=12$ $s=4.01$

($n \geq 30$ ✓)

$$\begin{aligned} 99\% \text{ UCB for } \mu &: \bar{x} + z_{\alpha} \frac{s}{\sqrt{n}} \\ &= 12 + 2.326 \left(\frac{4.01}{\sqrt{80}} \right) \\ &= 13.04 \end{aligned}$$

$$\boxed{\mu \leq 13.04}$$

② $n=45$ $\bar{x}=89$ $s=6.52$

($n \geq 30$ ✓)

$$\begin{aligned} 95\% \text{ LCB for } \mu &: \bar{x} - z_{\alpha} \frac{s}{\sqrt{n}} \\ &= 89 - 1.645 \left(\frac{6.52}{\sqrt{45}} \right) \\ &= 87.40 \end{aligned}$$

$$\boxed{87.40 \leq \mu}$$

$$\textcircled{3} \quad n=300 \quad \hat{p}=0.025 \\ \hat{q}=1-\hat{p}=0.975$$

$$(n\hat{p} > 5 \text{ and } n\hat{q} > 5)$$

$$\begin{aligned} 90\% \text{ UCB for } p: \quad & \hat{p} + z_{\alpha} \sqrt{\frac{\hat{p}\hat{q}}{n}} \\ & = 0.025 + 1.282 \sqrt{\frac{0.025(0.975)}{300}} \\ & \approx 0.037 \end{aligned}$$

$$\boxed{p \leq 0.037}$$

$$\textcircled{4} \quad n=40 \quad \bar{x}=25 \quad s=5$$

$$(n \geq 30)$$

$$\begin{aligned} 95\% \text{ UCB for } \mu: \quad & \bar{x} + z_{\alpha} \frac{s}{\sqrt{n}} \\ & = 25 + 1.645 \left(\frac{5}{\sqrt{40}} \right) \\ & \approx 26.300 \end{aligned}$$

$$\boxed{\mu \leq 26.300}$$

$$(5) \quad n=40 \quad \bar{x}=25 \quad s=5$$

$$(n \geq 30)$$

$$\begin{aligned} 99\% \text{ LCB for } \mu: \quad & \bar{x} - z_{\alpha} \frac{s}{\sqrt{n}} \\ & = 25 - 2.326 \left(\frac{5}{\sqrt{40}} \right) \\ & \approx 23.161 \end{aligned}$$

$$\boxed{23.161 \leq \mu}$$

$$(6) \quad n=1100 \quad x=16$$

$$\hat{p} = \frac{16}{1100} \quad \hat{q} = 1 - \hat{p} = \frac{1084}{1100}$$

$$(n\hat{p} > 5 \text{ and } n\hat{q} > 5)$$

$$\begin{aligned} 98\% \text{ UCB for } p: \quad & \hat{p} + z_{\alpha} \sqrt{\frac{\hat{p}\hat{q}}{n}} \\ & = \frac{16}{1100} + 2.054 \sqrt{\frac{16(1084)}{1100^3}} \\ & \approx 0.022 \end{aligned}$$

$$\boxed{p \leq 0.022}$$

⑦

$$\sigma = 0.02$$

$$n \geq ?$$

$$98\% \text{ CI for } \mu: \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$\begin{array}{c} \text{-----} \\ \left[\text{-----} \right] \\ \text{-----} \\ \underbrace{\hspace{1.5cm}}_{z_{\alpha/2} \frac{\sigma}{\sqrt{n}}} \quad \underbrace{\hspace{1.5cm}}_{z_{\alpha/2} \frac{\sigma}{\sqrt{n}}} \\ \underbrace{\hspace{3cm}}_{2z_{\alpha/2} \frac{\sigma}{\sqrt{n}}} \end{array}$$

$$\text{Want } 2z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < 0.003$$

$$2(2.326) \frac{(0.02)}{\sqrt{n}} < 0.003$$

$$\frac{2(2.326)(0.02)}{0.003} < \sqrt{n}$$

$$\left[\frac{2(2.326)(0.02)}{0.003} \right]^2 < n$$

$$961.8 \dots < n$$

$$n \geq 962$$

because n is an integer

$$\textcircled{8} \quad \hat{p} = 0.136 \quad n \geq ? \quad \hat{q} = 1 - \hat{p} = 0.864$$

$$99\% \text{ CI for } p: \quad \hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$\left[\underbrace{\hat{p}}_{z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}} \pm \underbrace{z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}} \right]$$

$$2 z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$\text{Want } 2 z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} < 0.2$$

$$2(2.576) \frac{\sqrt{0.136(0.864)}}{\sqrt{n}} < 0.2$$

$$\frac{2(2.576) \sqrt{0.136(0.864)}}{0.2} < \sqrt{n}$$

$$\left[\frac{2(2.576) \sqrt{0.136(0.864)}}{0.2} \right]^2 < n$$

$$n > 77.97\dots$$

$n \geq 78$ because n is
an integer

⑨ 95% CI for p :

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

margin of Error

Want $z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} < 0.04$

When \hat{p} and \hat{q} are unknown use
 $\hat{p} = 0.5$ and $\hat{q} = 0.5$

$$\frac{1.96 \sqrt{0.5(0.5)}}{\sqrt{n}} < 0.04$$

$$\frac{1.96 \sqrt{0.5(0.5)}}{0.04} < \sqrt{n}$$

$$\left[\frac{1.96 \sqrt{0.5(0.5)}}{0.04} \right]^2 < n$$

$$n > 600.25$$

$$n \geq 601$$

because n is an integer

(10) 95% CI for p :

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

↑ Margin of Error

Want $z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} < 0.03$

When \hat{p} and \hat{q} are unknown
use $\hat{p} = 0.5$ and $\hat{q} = 0.5$

$$\frac{1.96 \sqrt{0.5/0.5}}{\sqrt{n}} < 0.03$$

$$\frac{1.96 \sqrt{0.5/0.5}}{0.03} < \sqrt{n}$$

$$\left[\frac{1.96 \sqrt{0.5/0.5}}{0.03} \right]^2 < n$$

$$n > 1067.11 \dots$$

$$n \geq 1068$$

because n is an integer