Solutions

(1) $n = 50 \quad \bar{x} = 38 \quad s = 3.32$

$(n \geq 30)$ ✓

98% CI for μ: $\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$

$$38 \pm (2.326) \frac{3.32}{\sqrt{50}}$$

$$\boxed{36.91 \leq \mu \leq 39.09}$$

(2) $n_1 = 35 \quad \bar{x}_1 = 12 \quad s_1 = 1.11 \quad n_2 = 40 \quad \bar{x}_2 = 15 \quad s_2 = 1.89$

$(n_1 \geq 30$ and $n_2 \geq 30)$ ✓

95% CI for $\mu_1 - \mu_2$: $\bar{x}_1 - \bar{x}_2 \pm z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

$$12 - 15 \pm 1.96 \sqrt{\frac{(1.11)^2}{35} + \frac{(1.89)^2}{40}}$$

$$\boxed{-3.69 \leq \mu_1 - \mu_2 \leq -2.31}$$

③ $n=600 \quad \hat{p}=0.04 \qquad \hat{q}=1-\hat{p}=0.96$

$(n\hat{p}>5$ and $n\hat{q}>5)$

99% CI for p: $\quad \hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}}$

$$0.04 \pm 2.576\sqrt{\frac{0.04\times 0.96}{600}}$$

$$\boxed{0.02 \le p \le 0.06}$$

④ $n_1=100 \quad \hat{p}_1=0.06 \quad n_2=150 \quad \hat{p}_2=0.09$

$\hat{q}_1=1-\hat{p}_1=0.94 \qquad \hat{q}_2=1-\hat{p}_2=0.91$

$(n_1\hat{p}_1>5,\ n_1\hat{q}_1>5,\ n_2\hat{p}_2>5,\ n_2\hat{q}_2>5)$

98% CI for p_1-p_2: $\quad \hat{p}_1-\hat{p}_2 \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}_1\hat{q}_1}{n_1}+\dfrac{\hat{p}_2\hat{q}_2}{n_2}}$

$$0.06-0.09 \pm 2.326\sqrt{\frac{0.06(0.94)}{100}+\frac{0.09(0.91)}{150}}$$

$$\boxed{-0.11 \le p_1-p_2 \le 0.05}$$

(5) $\sigma = 0.02$ $n = 100$ $\bar{x} = 10.998$

Assumptions: $n \geq 30$ ✓

98% CI for μ: $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$$10.998 \pm 2.326 \left(\frac{0.02}{\sqrt{100}}\right)$$

$$\boxed{10.99 \leq \mu \leq 11.00}$$

(6) The 99% CI would be wider than the 98% CI because $z_{\alpha/2}$ is larger for $1-\alpha = 0.98$

Intuitively: We are more confident that μ lies in the 99% CI so this interval should be wider.

(7) $n = 500$ $x = 68$ $\hat{p} = \frac{68}{500}$

$$\hat{q} = 1 - \hat{p} = \frac{432}{500}$$

Assumptions: $n\hat{p} > 5$ and $n\hat{q} > 5$ ✓

95% CI for p: $\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$$\frac{68}{500} \pm 1.96 \sqrt{\frac{68(432)}{500^3}}$$

$$0.106 \leq p \leq 0.166$$

$$\frac{68}{500}\left(\frac{432}{500}\right)\frac{1}{500}$$

(8) $n_1 = 30$ $\bar{x}_1 = 15$ $s_1^2 = 16$ $n_2 = 40$ $\bar{x}_2 = 17$ $s_2^2 = 100$

Assumptions: $n_1 \geq 30$ and $n_2 \geq 30$ ✓

99% CI for $\mu_1 - \mu_2$: $\bar{x}_1 - \bar{x}_2 \pm z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

$$15 - 17 \pm 2.576 \sqrt{\frac{16}{30} + \frac{100}{40}}$$

$$-6.486 \leq \mu_1 - \mu_2 \leq 2.486$$

Conclusion: No significant evidence that one group performs faster.

(9) $n_1 = 180$ $x_1 = 126$ $n_2 = 110$ $x_2 = 54$

$\hat{p}_1 = \frac{126}{180}$ $\hat{q}_1 = 1 - \hat{p}_1 = \frac{54}{180}$ $\hat{p}_2 = \frac{54}{110}$ $\hat{q}_2 = 1 - \hat{p}_2 = \frac{56}{110}$

Assumptions: $n_1\hat{p}_1 > 5$, $n_1\hat{q}_1 > 5$, $n_2\hat{p}_2 > 5$, $n_2\hat{q}_2 > 5$ ✓

90% CI for $p_1 - p_2$: $\hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$

$$\frac{126}{180} - \frac{54}{110} \pm 1.645 \sqrt{\frac{126(54)}{180^3} + \frac{54(56)}{110^3}}$$

$$0.113 \leq p_1 - p_2 \leq 0.306$$

Conclusion: Significant evidence that $p_1 > p_2$.

(10) 95% of samples will produce a confidence interval that contains μ.