

Solutions

① a) $z = \frac{X - \mu}{\sigma} = \frac{212 - 150}{23} \approx 2.70$

b) $z = \frac{X - \mu}{\sigma} = \frac{116 - 150}{23} \approx -1.48$

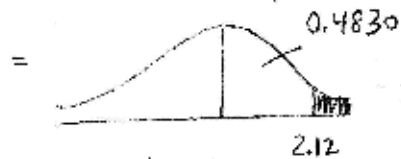
② a) $P(z < 1.63)$



= $0.5 + 0.4484$

= 0.9484

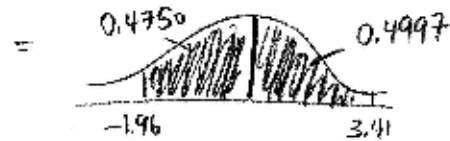
b) $P(z > 2.12)$



= $0.5 - 0.4830$

= 0.0170

c) $P(-1.96 \leq z \leq 3.41)$



= $0.4750 + 0.4997$

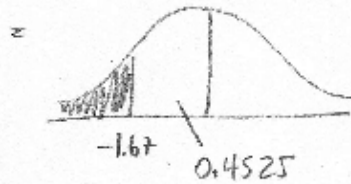
= 0.9747

③

a) $P(X < 230)$

= $P(Z < -1.67)$

$$z = \frac{230 - 250}{12} \approx -1.67$$



= $0.5 - 0.4525$

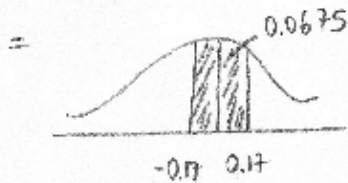
= 0.0475

b) $P(248 < X < 252)$

= $P(-0.17 < Z < 0.17)$

$$z_1 = \frac{248 - 250}{12} \approx -0.17$$

$$z_2 = \frac{252 - 250}{12} \approx 0.17$$



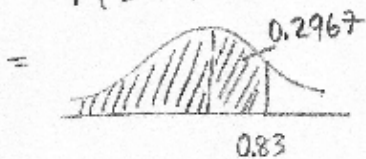
= 2×0.0675

= 0.1350

c) $P(X < 260)$

= $P(Z < 0.83)$

$$z = \frac{260 - 250}{12} \approx 0.83$$



= $0.5 + 0.2967$

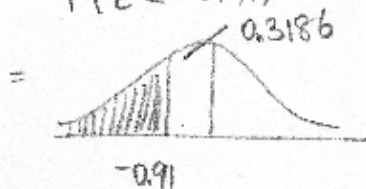
= 0.7967

④ Let $X = \text{weight}$ X is normal ✓

a) $P(X < 160)$

$$z = \frac{160 - 180}{22} \approx -0.91$$

$$= P(z < -0.91)$$



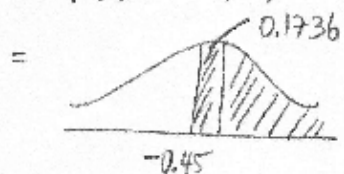
$$= 0.5 - 0.3186$$

$$= 0.1814$$

b) $P(X > 170)$

$$z = \frac{170 - 180}{22} \approx -0.45$$

$$= P(z > -0.45)$$



$$= 0.1736 + 0.5$$

$$= 0.6736$$

⑤ Let $X = \text{mass}$ X is normal ✓

$$\mu = 85 \quad \sigma^2 = 0.64$$

$$\text{So } \sigma = \sqrt{0.64} = 0.8$$

(Recall σ must be positive).

a) $P(84 \leq X \leq 86.2)$

$$= P(-1.25 \leq Z \leq 1.5)$$



$$z_1 = \frac{84 - 85}{0.8} = -1.25$$

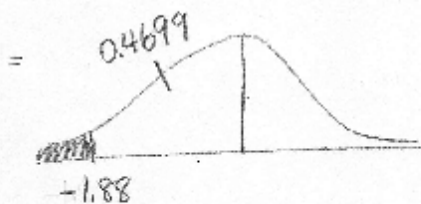
$$z_2 = \frac{86.2 - 85}{0.8} = 1.5$$

$$= 0.3944 + 0.4332$$

$$= 0.8276$$

b) $P(X < 83.5)$

$$= P(Z < -1.88)$$

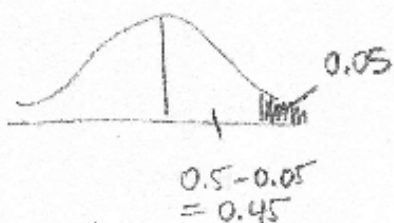


$$z = \frac{83.5 - 85}{0.8} \approx -1.88$$

$$= 0.5 - 0.4699$$

$$= 0.0301$$

⑥



Reverse look-up 0.45 :

$$z = 1.645$$

Now $z = \frac{X - \mu}{\sigma}$

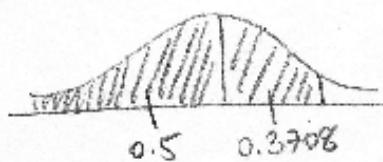
$$1.645 = \frac{X - 250}{12}$$

$$1.645(12) = X - 250$$

$$1.645(12) + 250 = X$$

$$X = 269.74$$

⑦



Reverse look-up 0.3708 :

$$z = 1.13$$

$$z = \frac{X - \mu}{\sigma}$$

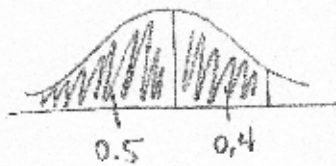
$$1.13 = \frac{X - 250}{12}$$

$$1.13(12) = X - 250$$

$$1.13(12) + 250 = X$$

$$X = 263.56$$

8 a)



Reverse look-up 0.4 :

$$z = 1.28$$

$$z = \frac{X - \mu}{\sigma}$$

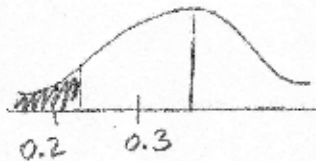
$$1.28 = \frac{X - 12}{0.34}$$

$$1.28(0.34) = X - 12$$

$$1.28(0.34) + 12 = X$$

$$X = 12.4352$$

b)



Reverse look-up 0.3 :

$$z = 0.84$$

Take $z = -0.84$ based on diagram.

$$z = \frac{X - \mu}{\sigma}$$

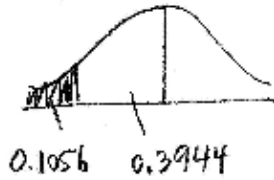
$$-0.84 = \frac{X - 12}{0.34}$$

$$-0.84(0.34) = X - 12$$

$$-0.84(0.34) + 12 = X$$

$$X = 11.7144$$

9



$$P(X < 9) = 0.1056$$

Reverse look-up 0.3944:

$$z = 1.25$$

Take $z = -1.25$ based on diagram.

So $X = 9$ corresponds to $z = -1.25$. Also $\mu = 10$

$$z = \frac{X - \mu}{\sigma}$$

$$-1.25 = \frac{9 - 10}{\sigma}$$

$$-1.25 = \frac{-1}{\sigma}$$

$$\sigma = \frac{-1}{-1.25}$$

$$\sigma = 0.8$$

10

$$P(X > 20) = 0.4$$



Reverse look-up 0.1 : $z = 0.25$

$X = 20$ corresponds to $z = 0.25$

$$z = \frac{X - \mu}{\sigma}$$

$$0.25 = \frac{20 - \mu}{\sigma} \quad (1)$$

$$P(X > 30) = 0.25$$



Reverse look-up 0.25 : $z = 0.67$

$X = 30$ corresponds to $z = 0.67$

$$z = \frac{X - \mu}{\sigma}$$

$$0.67 = \frac{30 - \mu}{\sigma} \quad (2)$$

$$(1): 0.25\sigma = 20 - \mu \quad (3)$$

$$(2): 0.67\sigma = 30 - \mu \quad (4)$$

$$(4) - (3): 0.42\sigma = 10$$

$$\sigma = \frac{10}{0.42} \approx 23.81$$

$\mu \approx 14.05$
$\sigma \approx 23.81$

$$\rightarrow (3) \quad 0.25(23.81) = 20 - \mu$$

$$\mu = 20 - 0.25(23.81) \approx 14.05$$