Solutions

(1) $P(X \leq 2)$

$= P(X=0) + P(X=1) + P(X=2)$

$= \frac{4^0 e^{-4}}{0!} + \frac{4^1 e^{-4}}{1!} + \frac{4^2 e^{-4}}{2!}$

$= e^{-4}\left(\frac{4^0}{0!} + \frac{4^1}{1!} + \frac{4^2}{2!}\right)$

≈ 0.238

(2) $P(X > 3)$

$= 1 - P(X \leq 3)$.

$= 1 - \left[P(X=0) + P(X=1) + P(X=2) + P(X=3)\right]$

$= 1 - \left[\frac{5^0}{0!}e^{-5} + \frac{5^1}{1!}e^{-5} + \frac{5^2}{2!}e^{-5} + \frac{5^3}{3!}e^{-5}\right]$

$= 1 - e^{-5}\left(\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!}\right)$

≈ 0.735

(3) $P(2 \leq X \leq 6)$

$= P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$= e^{-2.6}\left[\frac{(2.6)^2}{2!} + \frac{(2.6)^3}{3!} + \frac{(2.6)^4}{4!} + \frac{(2.6)^5}{5!} + \frac{(2.6)^6}{6!}\right]$

≈ 0.715

(4)

X	$P(X)$
0	$P(X=0) = \frac{1.5^0}{0!} e^{-1.5} \approx 0.22$
1	0.33
2	0.25
3	0.13
4	0.05
5	0.01

(5) Let X = # typos on a given page.
X is a Poisson random variable with

$$\mu = \frac{400}{1000} = 0.4$$

a) $P(X=2) = \frac{(0.4)^2}{2!} e^{-0.4} \approx 0.05$

b)
$$\begin{aligned} P(X>1) &= 1 - P(X \le 1) \\ &= 1 - [P(X=0) + P(X=1)] \\ &= 1 - e^{-0.4}\left[\frac{(0.4)^0}{0!} + \frac{(0.4)^1}{1!}\right] \\ &\simeq 0.06 \end{aligned}$$

(6) Let X = # server requests in the next hour

X is a Poisson random variable with

$\mu = 12$

$$P(X \le 4) = P(X=0) + P(X=1) + \ldots + P(X=4)$$

$$= e^{-12}\left[\frac{(12)^0}{0!} + \frac{(12)^1}{1!} + \ldots + \frac{(12)^4}{4!}\right]$$

$$\approx 0.008$$

(7) Let X = # server requests in the next 15 minutes.

$$\frac{12}{60\text{ min}} = \frac{3}{15\text{ min}}$$

X is a Poisson random variable with $\mu = 3$.

$$P(X \le 1) = P(X=0) + P(X=1)$$

$$= \frac{3^0}{0!}e^{-3} + \frac{3^1}{1!}e^{-3}$$

$$\approx 0.199$$

(8) $P(X=0) = 0.6$

$$\frac{\mu^0 e^{-\mu}}{0!} = 0.6$$

$$\boxed{e^{-\mu} = 0.6}$$

Now taking ln of both sides :

$$-\mu = \ln 0.6$$

$$\boxed{\mu = -\ln 0.6}$$

Now $P(X=3) = \dfrac{\mu^3 e^{-\mu}}{3!}$

$$= \frac{(-\ln 0.6)^3 (0.6)}{3!}$$

$$\approx 0.013$$

(9)

X	P(x)
0	$P(X=0) = \frac{5^0}{0!} e^{-5} \approx 0.007$
1	0.034
2	0.084
3	0.140
4	0.175

Summing these, $P(X \leq 3) \approx 0.265$

Now including $P(X=4)$ in the sum,

$$P(X \leq 4) \approx 0.440$$

So the minimum value of k is $k = 4$.

(10) $P(X=0) + P(X=1) + P(X=2) + \ldots$

$$= \frac{\mu^0}{0!}e^{-\mu} + \frac{\mu^1}{1!}e^{-\mu} + \frac{\mu^2}{2!}e^{-\mu} + \ldots$$

$$= e^{-\mu}\left[\frac{\mu^0}{0!} + \frac{\mu^1}{1!} + \frac{\mu^2}{2!} + \ldots\right]$$

$$= e^{-\mu}\left(e^{\mu}\right)$$

$$= 1$$