# Solutions

① a) $20C6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^{14} \approx 0.18$

b) $$P(5 \le X \le 7) = 20C5 \left(\tfrac{1}{3}\right)^5 \left(\tfrac{2}{3}\right)^{15} + 20C6 \left(\tfrac{1}{3}\right)^6 \left(\tfrac{2}{3}\right)^{14} + 20C7 \left(\tfrac{1}{3}\right)^7 \left(\tfrac{2}{3}\right)^{13}$$
$$\approx 0.51$$

c) $$P(x \le 17) = 1 - P(x \ge 18)$$
$$= 1 - \left[20C20 \left(\tfrac{1}{3}\right)^{20} \left(\tfrac{2}{3}\right)^0 + 20C19 \left(\tfrac{1}{3}\right)^{19} \left(\tfrac{2}{3}\right)^1 + 20C18 \left(\tfrac{1}{3}\right)^{18} \left(\tfrac{2}{3}\right)^2\right]$$
$$\simeq 1$$

d) e) The histogram would be roughly mound-shaped because with $n = 20$, $p = \frac{1}{3}$, $q = \frac{2}{3}$ we have both $\boxed{np > 5 \text{ and } nq > 5}$.

② $n = 4$ (# of trials) $\quad p = P(5) = \frac{1}{6} \quad q = 1 - p = \frac{5}{6}$

$$P(x \ge 1) = 1 - P(x = 0)$$
$$= 1 - 4C0 \left(\tfrac{1}{6}\right)^0 \left(\tfrac{5}{6}\right)^4$$
$$\simeq 0.518$$

③ $n=6$ $p=P(4)=\frac{1}{6}$ $q=1-p=\frac{5}{6}$

\# of trials

$$P(x \le 1) = P(x=0) + P(x=1)$$

$$= 6C0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^6 + 6C1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^5$$

$$\simeq 0.737$$

④ $n=8$ $p=P(H)=\frac{1}{2}$ $q=1-p=\frac{1}{2}$

$$\underbrace{P(\geq 5 \text{ heads})}_{\text{more heads than tails}} = P(x=5) + P(x=6) + P(x=7) + P(x=8)$$

$$= 8C5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^3 + 8C6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2 + 8C7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^1 + 8C8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^0$$

$$\approx 0.363$$

⑤ $q=1-p=0.9$

| X | P(x) |
|---|---|
| 0 | $4C0(0.1)^0(0.9)^4 = 0.6561$ |
| 1 | $4C1(0.1)^1(0.9)^3 = 0.2916$ |
| 2 | $4C2(0.1)^2(0.9)^2 = 0.0486$ |
| 3 | $4C3(0.1)^3(0.9)^1 = 0.0036$ |
| 4 | $4C4(0.1)^4(0.9)^0 = 0.0001$ |

(6) From calculator

$\mu = 0.4$

$\sigma^2 = 0.36$

For any binomial random variable

$\mu = np$

$\sigma^2 = npq$

(7) This is a binomial experiment.

$n = 8 \quad p = P(\text{club}) = \frac{1}{4} \quad q = 1 - p = \frac{3}{4}$

$P(X=3) = {}_8C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^5 \approx 0.208$

(8) This is not a binomial experiment.
P(club) varies as cards are removed from the deck.

Select 8 cards in sequence $\quad n(S) = {}_{52}P_8$

| | | | | | |
|---|---|---|---|---|---|
| $n(A) =$ | ${}_{13}C_3$ | $\times$ | ${}_{39}C_5$ | $\times$ | ${}_8P_8$ |
| | Choose 3 clubs | and | Choose 5 non-clubs | and | order the 8 cards |

$$P(A) = \frac{{}_{13}C_3 \times {}_{39}C_5 \times {}_8P_8}{{}_{52}P_8} \approx 0.219$$

(9) a) This is not a binomial experiment because P(Liberal) varies slightly as more voters are contacted. But since the population is large, the experiment is approximately binomial.

P(exactly 38 voted Liberal)

$\approx 200C38\,(0.189)^{38}(0.811)^{162}$

$\approx 0.072$

$n = 200$
$p = 0.189$
$q = 1 - p = 0.811$

b) Need $\frac{\text{population size}}{n} \geq 20$

$\frac{14.6 \text{ million}}{200} \geq 20$ ✓

(10) 4 non-♡ and 1 ♡ in any order then 1 ♡

$P(♡) = \frac{1}{4}$
$P(\text{non-}♡) = \frac{3}{4}$

Probability = $5C1 \times \left(\frac{3}{4}\right)^4 \times \left(\frac{1}{4}\right)^2$

\# of possible orders — P(4 non-♡) — P(2 ♡)

$\approx 0.099$