

## Solutions

① a)  $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.16}{0.4} = 0.4$

b)  $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.16}{0.7} \approx 0.23$

Note:  $B \cap A$  is the same set as  $A \cap B$ .

② No. Three ways to check:

i)  $P(A|B) \neq P(A)$

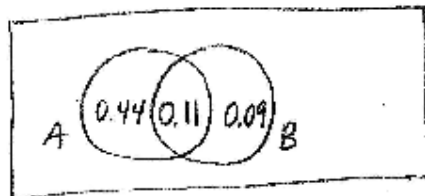
ii)  $P(B|A) \neq P(B)$

iii)  $P(A)P(B) \neq P(A \cap B)$

You only need to check one of these

③ Since A and B are independent

$$P(A \cap B) = P(A)P(B) = 0.11$$



$$P(A \cup B) = 0.44 + 0.11 + 0.09 = 0.64$$

$$\textcircled{4} \quad n(S) = 2 \times 2 \times 2 = 8$$

$$E = \{TTT, TTH, THT, HTT\}$$

$$F = \{HHT, HTH, THH, TTH, THT, HTT\}$$

$$P(E) = \frac{4}{8} = 0.5$$

$$P(E|F) = \frac{n(E \cap F)}{n(F)}$$

$$= \frac{3}{6}$$

$$= 0.5$$

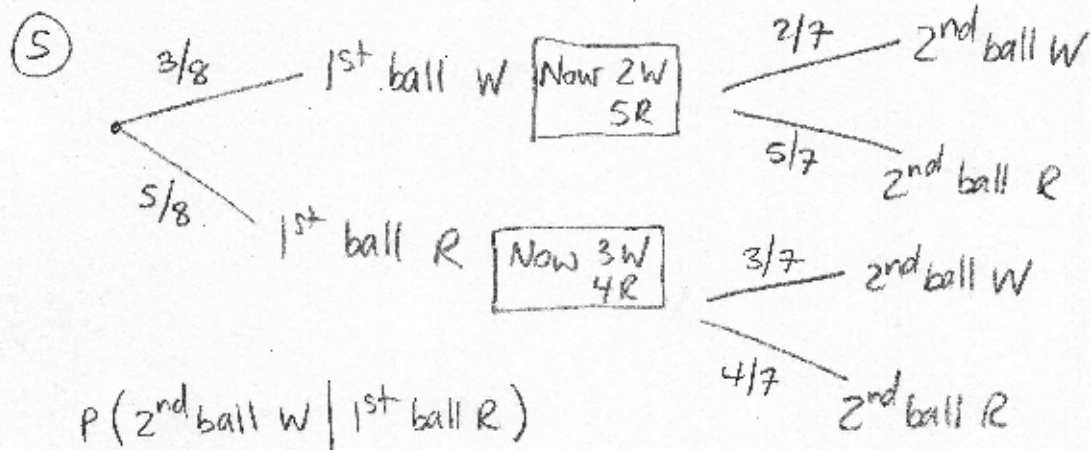
$$E \cap F = \{TTH, THT, HTT\}$$

events, Yes, E and F are independent events, because  $P(E) = P(E|F)$ .

Alternatively, check that

$$P(F) = P(F|E)$$

$$\text{or } P(E \cap F) = P(E)P(F).$$



$$P(\text{2nd ball W} \mid \text{1st ball R})$$

$$= 3/7$$

from tree diagram

- ⑥
- A : ace or heart  
 B : not a club

$$P(A|B) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{15}{39} \approx 0.38$$

$$n(A) = 16$$

$$n(B) = 39 \approx 0.38 \text{ (not a club)}$$

$$A \cap B = \{A \heartsuit, 2 \heartsuit, \dots, K \heartsuit, A \diamond, A \clubsuit\}$$

$$n(A \cap B) = 15$$

$$n(B) = 39$$

⑦

$$P(A) = \frac{13}{52} = 0.25$$

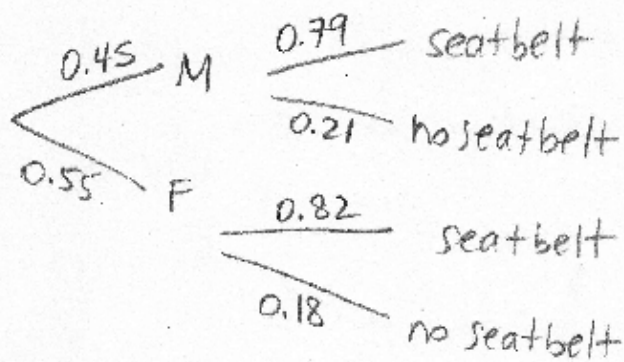
$$P(A|B) = \frac{n(A \cap B)}{n(B)} = \frac{12}{48} = 0.25$$

Yes, A and B are independent because

$$P(A) = P(A|B).$$

Alternatively: Compare  $P(B)$  and  $P(B|A)$   
 or  $P(A \cap B)$  and  $P(A)P(B)$

8



a) 
$$\begin{aligned} P(\text{seatbelt}) &= P(M \cap \text{seatbelt}) + P(F \cap \text{seatbelt}) \\ &= 0.45(0.79) + 0.55(0.82) \\ &= 0.8065 \end{aligned}$$

b) 
$$\begin{aligned} P(M \text{ and no seatbelt}) &= 0.45(0.21) \\ &= 0.0945 \end{aligned}$$

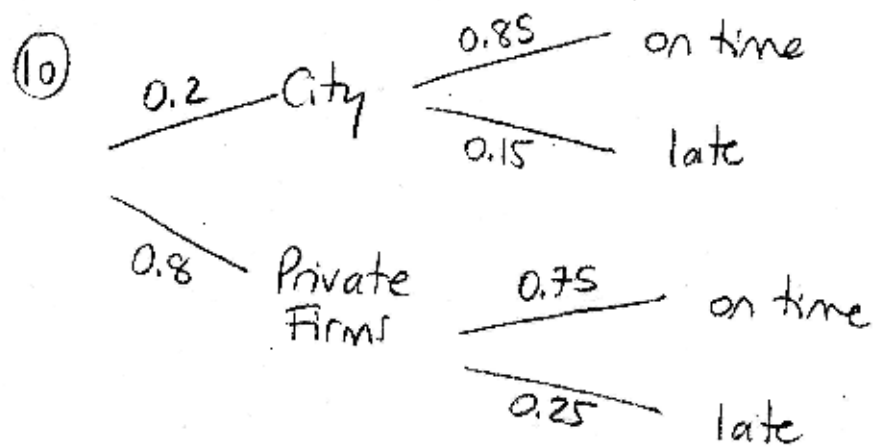
c) 
$$P(\text{no seatbelt} | F) = 0.18$$

9) a) 
$$P(\text{Powerpoint}) = 0.26 + 0.20 = 0.46$$

b) 
$$P(\text{successful}) = 0.26 + 0.36 = 0.62$$

c) 
$$P(\text{Keynote} | \text{successful}) = \frac{0.36}{0.62} \approx 0.58$$

d) 
$$P(\text{unsuccessful} | \text{Powerpoint}) = \frac{0.20}{0.46} \approx 0.43$$



a)  $P(\text{City and on time})$   
 $= 0.2(0.85)$   
 $= 0.17$

b)  $P(\text{on time}) = P(\text{City and on time})$   
 $+ P(\text{Private and on time})$   
 $= 0.2(0.85) + 0.8(0.75)$   
 $= 0.77$

c)  $P(\text{late} | \text{Private firm}) = 0.25$