Stat 254 Assignment Two (Q4)

Name:______________________________

Assignments must be completed on this paper. Marks may be deducted for not showing all your work.

1. [4 marks] Consider the random variable X with the following probability density function: $f(x) = \frac{1}{(7\ln 2)x}$ for $4 \le x \le 512$ and $f(x) = 0$ otherwise.

a) Find the expected value of X. Give an exact value.

$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{-\infty}^{4} 0\,dx + \int_{4}^{512} \frac{1}{7\ln 2}\,dx + \int_{512}^{\infty} 0\,dx$$

$$= \left[\frac{x}{7\ln 2}\right]_{4}^{512}$$

$$= \frac{508}{7\ln 2}$$

b) Find $P(50 \le X \le 80)$. Round your answer to two decimal places.

$$P(50 \le X \le 80) = \int_{50}^{80} f(x)\,dx$$

$$= \int_{50}^{80} \frac{1}{(7\ln 2)x}\,dx$$

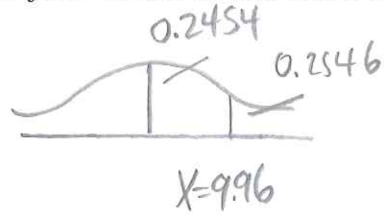
$$= \left[\frac{\ln x}{7\ln 2}\right]_{50}^{80}$$

$$= \frac{\ln 80}{7\ln 2} - \frac{\ln 50}{7\ln 2}$$

$$\approx 0.10$$

2. [4 marks] The diameters of ball bearings at a manufacturing plant are normally distributed with mean μ and SD σ. Over a long period of time it is observed that 25.46% of ball bearings have diameters more than 9.96 mm, and 7.78% of ball bearings have diameters more than 10.05 mm. Find μ and σ. Round your values to one decimal place.

X = diameter $\boxed{\text{Normal}}$



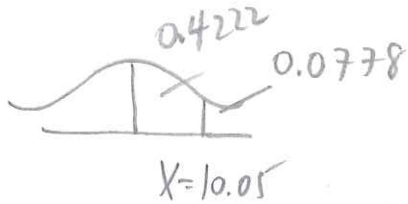
Reverse look-up area = 0.2454

$z = 0.66$

$\rightarrow z = \dfrac{X-\mu}{\sigma}$

$0.66 = \dfrac{9.96-\mu}{\sigma}$ 1

or $\mu + 0.66\sigma = 9.96$ (1)



Reverse look-up area = 0.4222

$z = 1.42$

$\rightarrow z = \dfrac{X-\mu}{\sigma}$

$1.42 = \dfrac{10.05-\mu}{\sigma}$ 1

or $\mu + 1.42\sigma = 10.05$ (2)

(2)-(1): $0.76\sigma = 0.09$ 2

$\boxed{\sigma = 0.1 \text{ mm}}$ $\rightarrow$ (1) $\boxed{\mu = 9.9 \text{ mm}}$

3. [4 marks] Each shipment ordered from a retail website has a 12% probability of being delayed. Estimate the probability of between 90 and 110 delays (inclusive) among 850 shipments.

Binomial $n = 850$ $p = P(\text{delay}) = 0.12$ $q = 1 - p = 0.88$ 1

$X = \#$ delays

$P(90 \le X \le 110)$ $\longrightarrow$

$np, nq > 5$ ✓

$\mu = np = 102$ 1

$\sigma = \sqrt{npq} = \sqrt{89.76}$

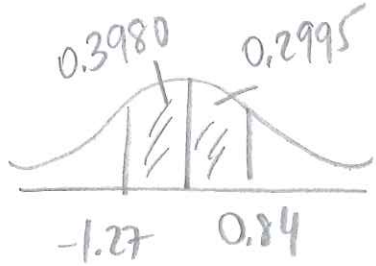
$z = \dfrac{X - \mu}{\sigma}$

$z_1 = \dfrac{90 - 102}{\sqrt{89.76}} \approx -1.27$

$z_2 = \dfrac{110 - 102}{\sqrt{89.76}} \approx 0.84$

1 $= P(-1.27 \le z \le 0.84)$

$=$



1 $= 0.6975$

Alternatively: $P\left(\dfrac{90}{850} \le \hat{p} \le \dfrac{110}{850}\right)$

using Central Limit Theorem

$z = \dfrac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$

4. [3 marks] As part of their job interview process, 900 aspiring engineers write a standardized test which is scored out of 100. The mean test score is 71 with a variance of 25. A random sample of 60 tests is selected. Find the probability that the mean of the sampled test scores is less than 72.

$\boxed{\mu = 71}$ $\quad \sigma^2 = 25 \quad$ $n = 60$

$\boxed{\sigma = 5}$

$P(\bar{x} < 72) \longrightarrow$

$n \geq 30$ ✓

$z = \dfrac{\bar{x} - \mu}{(\sigma/\sqrt{n})}$ 1

$= \dfrac{72 - 71}{(5/\sqrt{60})}$

$\simeq 1.55$ 1

$= P(z < 1.55)$

$=$

$= 0.9394$ 1

5. [3 marks] A website tracks how long visitors stay on the site. A sample of 82 visitors stayed for an average of 1.33 minutes, with a standard deviation of 0.49 minutes. Find a 95% upper confidence bound for the average amount of time visitors stay on the site. Round your answer to two decimal places.

$n = 82 \quad \bar{x} = 1.33 \quad s = 0.49$

95% UCB for μ:

$\bar{x} + z_\alpha\left(\dfrac{s}{\sqrt{n}}\right)$

$1 - \alpha = 0.95$

$z_\alpha = 1.645$ 1

$= 1.33 + 1.645\left(\dfrac{0.49}{\sqrt{82}}\right)$ 1

$\simeq 1.42$

$\boxed{\mu \leq 1.42 \text{ minutes}}$ 1

6. [7 marks] Test whether the population proportions p_1 and p_2 are equal at the 1% significance level given the following sample data:
$n_1 = 200, \hat{p}_1 = 0.81, n_2 = 400, \hat{p}_2 = 0.78$.

(-2) for $\hat{p}_1, \hat{p}_2$ here

a) State H_0 and H_a

2

$H_0: p_1 = p_2 \qquad H_a: p_1 \neq p_2 \qquad$ 2-tailed

b) Check any necessary assumptions.

1

$n_1\hat{p}_1, n_1\hat{q}_1, n_2\hat{p}_2, n_2\hat{q}_2 > 5 \qquad \left(\begin{array}{l} \hat{q}_1 = 1-\hat{p}_1 = 0.19 \\ \hat{q}_2 = 1-\hat{p}_2 = 0.22 \end{array}\right)$

c) Do you reject H_0? Show all your work.

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.81 - 0.78}{\sqrt{(0.79(0.21))\left(\frac{1}{200}+\frac{1}{400}\right)}} \simeq 0.85$$

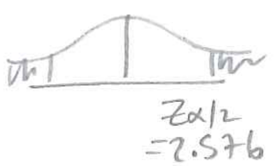
1

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2} = \frac{162 + 312}{600} = 0.79$$

(x = # that have the property)

1

$$\hat{q} = 1 - \hat{p} = 0.21$$



$\boxed{\text{Don't reject } H_0}$

$p_1 \simeq p_2$

1

d) Find the p-value.

$p =$ 

$= 2(0.5 - 0.3023)$
$= 0.3954$

1

7. [5 marks] Find the probability of making a Type II error in the hypothesis test below if the true value of μ is 11.1.

Test H_0: $\mu = 10.00$ at $\alpha = 0.05$ with $\overline{x} = 10.6, s = 1.8, n = 60$.

Want P(don't reject H_0) if $\mu = 11.1$

1) Non-rejection region:

$$\mu_0 \pm z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

$$10.00 \pm 1.960\left(\frac{1.8}{\sqrt{60}}\right)$$

$$9.54 \leq \overline{x} \leq 10.46$$

2

2) Find $P(9.54 \leq \overline{x} \leq 10.46)$ using $\mu = 11.1$

$= P(-6.71 \leq z \leq -2.75)$

$= 0.4970$

$= 0.5 - 0.4970$

$= 0.0030$

1

$$z = \frac{\overline{x} - \mu}{(s/\sqrt{n})}$$

$$z_1 = \frac{9.54 - 11.1}{(1.8/\sqrt{60})} \approx -6.71$$

$$z_2 = \frac{10.46 - 11.1}{(1.8/\sqrt{60})} \approx -2.75$$

2