Stat 254 Assignment One (Q4)

Name:______________________________

Assignments must be completed on this paper. Marks may be deducted for not showing all your work.

1. [2 marks] The data set below represents house prices in a certain city.

Price ($)	Relative Frequency
600,000	0.1
800,000	0.45
1,000,000	0.3
1,200,000	0.1
1,400,000	0.05

a) Find the mean

$\mu = (600{,}000)(0.1) + \dots + (1{,}400{,}000)(0.05)$
$= \$910{,}000$

b) Find the median

middle value, taking relative frequencies into account

median = $800,000

2. [4 marks] A data set has a mean of 121 and a standard deviation of 33. What can you say about the proportion of measurements that are more than 165?

$\mu = 121 \quad \sigma = 33$

$165 = \mu + k\sigma$

$165 = 121 + k(33)$

$44 = k(33)$

$k = 4/3$

1

$\geq 1 - \frac{1}{(\frac{4}{3})^2}$ of data lies in $\mu \pm \frac{4}{3}\sigma$

≥ 0.4375 " in $[77, 165]$ 1

≤ 0.5625 " outside $[77, 165]$

≤ 0.5625 " above 165

$\boxed{\text{At most } 56.25\% \text{ of measurements are } > 165}$ 2

3. [3 marks] Find the population variance for the following data set. Your answer will involve a:

$-a, -7, 3, a$

X	$X-\mu$	$(X-\mu)^2$
$-a$	$-a+1$	$(1-a)^2$
-7	-6	36
3	4	16
a	$a+1$	$(a+1)^2$

$\mu = \frac{-a-7+3+a}{4}$

$= -1$

$\sigma^2 = \frac{(1-a)^2 + 52 + (a+1)^2}{4}$

or $\frac{a^2+27}{2}$

4. [3 marks] In a class of 42 students, 23 have a part-time job and 17 have a car. Of the students who don't have a part-time job, 6 have a car. Find the probability that a student has:

a) a job and a car

	Job	No Job
Car	11	6
No Car	12	13

$P(\text{job and car}) = \frac{11}{42}$

b) neither a job nor a car

$P(\text{no job and no car}) = \frac{13}{42}$

5. [4 marks] A coin is tossed ten times. Find the probability that the first two tosses are tails or the last three tosses are heads. Round your answer to two decimal places.

$n(S) = 2 \times 2 \times \ldots = 2^{10}$

$\boxed{\text{Recall } n(A \cup B) = n(A) + n(B) - n(A \cap B)}$

$\# \text{outcomes} = \# TT\_\_\_\_ + \# \_\_\_\_HHH - \# TT\_\_\_\_HHH$

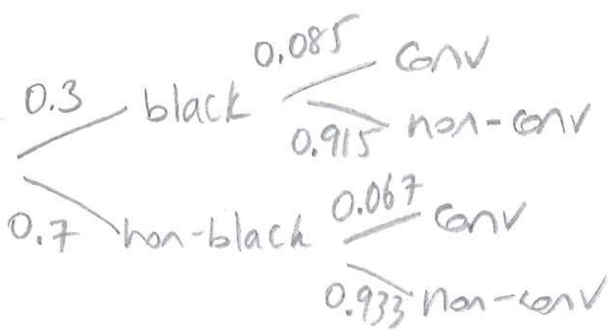
$= 2^8 + 2^7 - 2^5$

$\text{probability} = \dfrac{2^8 + 2^7 - 2^5}{2^{10}}$

≈ 0.34

6. [4 marks] Thirty percent of all cars are black. Of the black cars, 8.5% are convertibles. Of the non-black cars, 6.7% are convertibles. Find:

a) The probability that a black car is not a convertible



$P(\text{non-conv} \mid \text{black}) = 0.915$

1

b) The probability that a convertible is not black

$P(\text{non-black} \mid \text{conv}) = \dfrac{P(\text{non-black} \cap \text{conv})}{P(\text{conv})}$

$= \dfrac{0.7(0.067)}{[0.3(0.085) + 0.7(0.067)]}$

≈ 0.65

3

7. [4 marks] A shipment of 38 widgets contains 6 defective widgets (and 32 good widgets). Three widgets are randomly selected from the shipment. Let X be the number of defective widgets selected ~~selected~~.

a) Find the probability distribution of X. Round the probabilities to three decimal places.

X=# defective	Outcome	# Outcomes	P(X)
0	3G	32C3 = 4960	$\frac{4960}{8436} \approx 0.588$
1	2G and 1D	32C2×6C1 = 2976	0.353
2	1G and 2D	32C1×6C2 = 480	0.057
3	3D	6C3 = 20	0.002

1

2 8436

b) Find the expected value of X

$E(x) = 0(0.588) + 1(0.353) + 2(0.057) + 3(0.002)$

(also called μ) $= 0.473$ 1

8. [3 marks] An old plane has four engines which operate independently. Each engine operates correctly on 61% of flights. Find the probability that at least two of the four engines operate correctly on the plane's next flight. Round your answer to three decimal places.

Binomial $n=4$ $p=0.61$ $q=1-p=0.39$ 1

$X=$ # engines that operate correctly

$P(X \geq 2) = P(X=2)+P(X=3)+P(X=4)$ 1

$= 4C2(0.61)^2(0.39)^2 + 4C3(0.61)^3(0.39)$

$+ 4C4(0.61)^4(0.39)^0$

≈ 0.832 1

9. [3 marks] The average number of lightning strikes in a city is 1.7 per year. Find the probability distribution for the number of lightning strikes over the next two years. Round your probabilities to two decimal places. Ignore any probabilities that are less than 1%.

$\frac{1.7 \text{ strikes}}{\text{year}} = \frac{3.4 \text{ strikes}}{2 \text{ years}}$

$X=$ #strikes/2 years Use $\mu=3.4$

X	P(X)
0	$\frac{3.4^0 e^{-3.4}}{0!} \approx 0.03$
1	0.11
2	0.19
3	0.22
4	0.19
5	0.13
6	0.07
7	0.03
8	0.01