1. [5 marks] Find an **explicit** solution (solve for y):

$\frac{dy}{dx} + (1 + \frac{2}{x})y = 6$

Linear $\quad P(x) = 1 + \frac{2}{x}$

$$\int P(x)\,dx = x + 2\ln|x|$$

$$I.F. = e^{\int P(x)dx} = e^{x + 2\ln|x|}$$
$$= e^{x} \cdot e^{2\ln|x|}$$
$$= e^{x} \cdot e^{\ln|x|^2}$$
$$= |x|^2 e^{x}$$
$$= x^2 e^{x}$$

2

$$x^2 e^{x} \frac{dy}{dx} + (1 + \frac{2}{x}) x^2 e^{x} y = 6x^2 e^{x}$$

Integrate:

2 $\quad x^2 e^{x} y = (6x^2 - 12x + 12)e^{x} + C$

	D	I
+	$6x^2$	e^x
−	$12x$	
+	12	

1 Explicit: $\boxed{y = 6 - \frac{12}{x} + \frac{12}{x^2} + \frac{C}{x^2 e^{x}}}$

2. [5 marks] Find an **explicit** solution (solve for y):

$$\frac{e^y\,dy}{dx} = \frac{2\sin x}{(e^y+1)^6} \qquad y(0) = 0$$

Separable $\quad (e^y+1)^6 e^y\,dy = 2\sin x\,dx$

$$\int (e^y+1)^6 e^y\,dy = 2\int \sin x\,dx$$

$u = e^y+1$

$du = e^y\,dy$

$\int (e^y+1)^6 e^y\,dy = \int u^6\,du$

$= \frac{u^7}{7} + \text{constant}$

(3) $\quad \frac{(e^y+1)^7}{7} = -2\cos x + C_1$

$$\boxed{(e^y+1)^7 = -14\cos x + C_2}$$

Sub $x=0$, $y=0$: $\quad 2^7 = -14 + C_2$

$C_2 = 142$ $\quad$ (1)

$(e^y+1)^7 = -14\cos x + 142$

$e^y+1 = \sqrt[7]{142 - 14\cos x}$

$e^y = \sqrt[7]{142 - 14\cos x} - 1$

EXPLICIT $\quad y = \ln\left(\sqrt[7]{142-14\cos x} - 1\right)$ $\quad$ (1)

3. [4 marks] Find an **implicit** solution:

$$(4x^3e^{y^2} + y\cos xy + 12x^2)dx + (2x^4ye^{y^2} + x\cos xy)dy = 0$$

$M_y = 4x^3(2ye^{y^2}) + \cos xy - xy\sin xy$

$N_x = 4x^3(2ye^{y^2}) + \cos xy - xy\sin xy$

$M_y = N_x \Rightarrow$ DE is exact

$f = \int (4x^3e^{y^2} + y\cos xy + 12x^2)dx$

$= x^4e^{y^2} + \sin xy + 4x^3 + g(y)$

AND

$f = \int (x^4 \cdot 2ye^{y^2} + x\cos xy)dy$

$= x^4e^{y^2} + \sin xy + h(x)$

$g(y) = 0$
$h(x) = 4x^3$

Conclude: $f = x^4e^{y^2} + \sin xy + 4x^3$ 3

DE is exact

$\Rightarrow df = 0$

$\Rightarrow \int df = \int 0$

$\Rightarrow f = C$

$\boxed{x^4e^{y^2} + \sin xy + 4x^3 = C}$ 1

4. [6 marks] Make the substitution $y = ux$ to find an **implicit** solution. Eliminate any absolute values or logarithms from your answer.

$xy^3dy = (y^4 - x^4)dx$

$$\begin{cases} y = ux \\ dy = u\,dx + x\,du \end{cases}$$ 1

$$\cancel{x(u^3x^3)}(u\,dx + x\,du) = (u^4x^4 - x^4)dx$$
u^3x^4

$$u^4x^4\,dx + u^3x^5\,du = u^4x^4\,dx - x^4\,dx$$

$$u^3x^5\,du = -x^4\,dx$$

$$u^3\,du = \frac{-dx}{x}$$ Separable

$$\int u^3\,du = -\int\frac{dx}{x}$$ 1

$$\frac{u^4}{4} = -\ln|x| + C_1$$ 1

Recall $y = ux$
$u = y/x$: $$\frac{y^4}{4x^4} = -\ln|x| + C_1$$ 1

$e^{LS} = e^{RS}$: $$e^{\frac{y^4}{4x^4}} = e^{-\ln|x| + C_1}$$

$$e^{\frac{y^4}{4x^4}} = e^{\ln|x|^{-1}} \cdot e^{C_1}$$

$$e^{\frac{y^4}{4x^4}} = \frac{e^{C_1}}{|x|}$$ →

$$e^{\frac{y^4}{4x^4}} = \frac{\pm e^{C_1}}{x}$$

$$\boxed{e^{\frac{y^4}{4x^4}} = \frac{C}{x}}$$

2

or $\boxed{x e^{\frac{y^4}{4x^4}} = C}$

OR $\boxed{x^4 e^{\frac{y^4}{x^4}} = C_2}$