Math 252 X01

Test One

Time: 50 minutes

Total: 16 marks

Name: ______________________________

1. [3 marks] Solve for y given that $y = 1$ when $x = \frac{\pi}{2}$:
$\frac{dy}{dx} = y^2 + 1$

$$\frac{dy}{y^2+1} = dx \qquad \text{separable}$$

$$\int \frac{dy}{y^2+1} = \int dx$$

$$\arctan y = x + C$$

$y=1$, $x=\frac{\pi}{2}$:

$$\arctan 1 = \frac{\pi}{2} + C$$

$$\frac{\pi}{4} = \frac{\pi}{2} + C$$

$$C = -\frac{\pi}{4}$$

$$\arctan y = x - \frac{\pi}{4}$$

$$y = \tan\left(x - \frac{\pi}{4}\right)$$

2. [5 marks] Solve for y:
$x^7 \frac{dy}{dx} + 2x^6 y = 9x^7 e^{3x}$

$$\frac{dy}{dx} + \frac{2}{x} y = 9e^{3x} \qquad \text{Linear}$$

$$\text{I.F.} = e^{\int \frac{2}{x} dx} = e^{2\ln|x|} = e^{\ln x^2} = x^2$$

$$x^2 \frac{dy}{dx} + 2xy = 9x^2 e^{3x}$$

D	I
$+\ 9x^2$	e^{3x}
$-\ 18x$	$e^{3x}/3$
$+\ 18$	$e^{3x}/9$
	$e^{3x}/27$

$$x^2 y = 3x^2 e^{3x} - 2x e^{3x} + \frac{2}{3} e^{3x} + c$$

$$y = 3e^{3x} - \frac{2}{x} e^{3x} + \frac{2}{3x^2} e^{3x} + \frac{c}{x^2}$$

3. [5 marks] Find an implicit solution (eliminate any logarithms, absolute values and fractions):
$\frac{dy}{dt} = y^2 - 8y$

$$\frac{dy}{y^2-8y} = dt$$ Separable

Partial Fractions

$$\frac{1}{y(y-8)} = \frac{A}{y} + \frac{B}{y-8}$$

$$1 = A(y-8) + By$$

$y=0$: $1 = -8A \Rightarrow A = -\frac{1}{8}$

$y=8$: $1 = 8B \Rightarrow B = \frac{1}{8}$

$$\int \left(-\frac{1}{8}\frac{1}{y} + \frac{1}{8}\frac{1}{y-8}\right) dy = \int dt$$

$$-\frac{1}{8}\ln|y| + \frac{1}{8}\ln|y-8| = t + C_1$$

$$\ln|y| - \ln|y-8| = -8t + C_2$$

$$\ln\left|\frac{y}{y-8}\right| = -8t + C_2$$

$$\left|\frac{y}{y-8}\right| = \cancel{e^{-8t+C_2}} \quad C_3e^{-8t}$$

$$\frac{y}{y-8} = \cancel{\pm C_3 e^{-8t}} \quad Ce^{-8t}$$

$$y = Ce^{-8t}(y-8)$$

4. [3 marks] Find an implicit solution if $y = 0$ when $x = 1$:
$(2xe^{y^3} + 9\sec y + 4x^3)dx + (3x^2y^2e^{y^3} + 9x\sec y\tan y)dy = 0$

$$M_y = 6xy^2e^{y^3} + 9\sec y\tan y = N_x$$

DE is exact

$$f(x,y) = \int (2xe^{y^3} + 9\sec y + 4x^3)dx$$
$$= x^2e^{y^3} + 9x\sec y + x^4 + g(y)$$

AND

$$f(x,y) = \int (3x^2y^2e^{y^3} + 9x\sec y\tan y)dy$$
$$= x^2e^{y^3} + 9x\sec y + h(x)$$

$$\Rightarrow \quad f(x,y) = x^2e^{y^3} + 9x\sec y + x^4$$

Solution : $\quad f = C$

$$x^2e^{y^3} + 9x\sec y + x^4 = C$$

$y=0$, $x=1$:

$$1 + 9 + 1 = C$$
$$C = 11$$

$$x^2e^{y^3} + 9x\sec y + x^4 = 11$$