Math 262
Systems of DEs Practice Problems

1. Find the general solution of $\mathbf{X}'(t) = \mathbf{A}\mathbf{X}(t)$ if

   (a) $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$.

   (b) $\mathbf{A} = \begin{pmatrix} -1 & -4 \\ 1 & -1 \end{pmatrix}$

   (c) $\mathbf{A} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$

   (d) $\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$

   (e) $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix}$

   (f) $\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

   (g) $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$

   (h) $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -3 & 2 & 4 \end{pmatrix}$

2. Solve the initial value problem.

$$\mathbf{X}'(t) = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \mathbf{X}(t), \quad \mathbf{X}(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

3. Use the method of variation of parameters to find the general solution of the system

$$\mathbf{X}'(t) = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \mathbf{X}(t) + \begin{pmatrix} 2e^{-t} \\ 3t \end{pmatrix}$$

4. Solve the initial value problem

$$\mathbf{X}'(t) = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \mathbf{X}(t) + \begin{pmatrix} 2 \\ t \end{pmatrix}, \quad \mathbf{X}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

5. Solve the initial value problem.

$$\mathbf{X}'(t) = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix} \mathbf{X}(t), \quad \mathbf{X}(0) = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

Eigenvectors are not unique.
Your answer might still be correct even if it appears to be different than the one given.

Answers.

1. (a) $\mathbf{X}(t) = c_1 \begin{pmatrix}1\\2\end{pmatrix} e^{5t} + c_2 \begin{pmatrix}-1\\1\end{pmatrix} e^{-t}$

   (b) $\mathbf{X}(t) = \left[c_1 \begin{pmatrix}-2\sin 2t\\ \cos 2t\end{pmatrix} + c_2 \begin{pmatrix}2\cos 2t\\ \sin 2t\end{pmatrix}\right] e^{-t}$

   (c) $\mathbf{X}(t) = c_1 \begin{pmatrix}2\\1\end{pmatrix} e^{t} + c_2 \left[\begin{pmatrix}2\\1\end{pmatrix} t + \begin{pmatrix}1\\0\end{pmatrix}\right] e^{t}$

   (d) $\mathbf{X}(t) = c_1 \begin{pmatrix}1\\1\\1\end{pmatrix} e^{4t} + c_2 \begin{pmatrix}1\\-2\\1\end{pmatrix} e^{t} + c_3 \begin{pmatrix}1\\0\\-1\end{pmatrix} e^{-t}$

   (e) $\mathbf{X}(t) = c_1 \begin{pmatrix}2\\-3\\2\end{pmatrix} e^{t} + \left[c_2 \begin{pmatrix}0\\-\sin 2t\\ \cos 2t\end{pmatrix} + c_3 \begin{pmatrix}0\\ \cos 2t\\ \sin 2t\end{pmatrix}\right] e^{t}$

   (f) $\mathbf{X}(t) = c_1 \begin{pmatrix}1\\1\\1\end{pmatrix} e^{2t} + \left[c_2 \begin{pmatrix}-1\\1\\0\end{pmatrix} + c_3 \begin{pmatrix}-1\\0\\1\end{pmatrix}\right] e^{-t}$

   (g) $\mathbf{X}(t) = c_1 \begin{pmatrix}-3\\4\\2\end{pmatrix} e^{-t} + c_2 \begin{pmatrix}0\\1\\-1\end{pmatrix} e^{2t} + c_3 \left[\begin{pmatrix}0\\1\\-1\end{pmatrix} t + \begin{pmatrix}1\\1\\0\end{pmatrix}\right] e^{2t}$

   (h) $\mathbf{X}(t) = c_1 \begin{pmatrix}0\\1\\-1\end{pmatrix} e^{2t} + c_2 \left[\begin{pmatrix}0\\1\\-1\end{pmatrix} t + \begin{pmatrix}1\\1\\0\end{pmatrix}\right] e^{2t} + c_3 \left[\begin{pmatrix}0\\1\\-1\end{pmatrix} \frac{t^2}{2} + \begin{pmatrix}1\\1\\0\end{pmatrix} t + \begin{pmatrix}2\\3\\0\end{pmatrix}\right] e^{2t}$

2. $\mathbf{X}(t) = \frac{5}{2} \begin{pmatrix}1\\1\end{pmatrix} e^{4t} - \frac{1}{2} \begin{pmatrix}1\\3\end{pmatrix} e^{2t} = \begin{pmatrix}\frac{5}{2}e^{4t} - \frac{1}{2}e^{2t}\\ \frac{5}{2}e^{4t} - \frac{3}{2}e^{2t}\end{pmatrix}$

3. $\mathbf{X}(t) = c_1 \begin{pmatrix}1\\-1\end{pmatrix} e^{-3t} + c_2 \begin{pmatrix}1\\1\end{pmatrix} e^{-t} + \begin{pmatrix}1\\1\end{pmatrix} te^{-t} + \frac{1}{2} \begin{pmatrix}1\\-1\end{pmatrix} e^{-t} + \begin{pmatrix}1\\2\end{pmatrix} t - \frac{1}{3} \begin{pmatrix}4\\5\end{pmatrix}$

4. $\mathbf{X}(t) = \begin{pmatrix}2\\2\end{pmatrix} e^{t} + \begin{pmatrix}2\\6\end{pmatrix} e^{-t} + \begin{pmatrix}1\\2\end{pmatrix} t - \begin{pmatrix}4\\7\end{pmatrix} = \begin{pmatrix}2e^{t} + 2e^{-t} + t - 4\\ 2e^{t} + 6e^{-t} + 2t - 7\end{pmatrix}$

5. $\mathbf{X}(t) = \begin{pmatrix}2\\-3\\2\end{pmatrix} e^{t} + 4e^{t} \begin{pmatrix}0\\ \cos 2t\\ \sin 2t\end{pmatrix} + 2e^{t} \begin{pmatrix}0\\ \sin 2t\\ -\cos 2t\end{pmatrix} = \begin{pmatrix}2e^{t}\\ -3e^{t} + 4e^{t}\cos 2t + 2e^{t}\sin 2t\\ 2e^{t} + 4e^{t}\sin 2t - 2e^{t}\cos 2t\end{pmatrix}$