

Math 252 Final Exam Review

1. Find an implicit solution. Don't simplify the solution.  
 $(y + y \cos xy)dx + (x + x \cos xy + \frac{2}{y})dy = 0$

Answer:

$$xy + \sin xy + 2 \ln |y| = C$$

2. Find an implicit solution. Don't simplify the solution.  
 $y' + \frac{y}{x} = x^3y^3$

Answer:

$$x^{-2}y^{-2} = -x^2 + C$$

3. Find an explicit solution.  
 $(x^2 + y^2)dx - xydy = 0$

Answer:

$$y = \pm \sqrt{2x^2 \ln |x| + Cx^2}$$

4. A car has mass  $m$ . The car's mass times acceleration is numerically proportional to its negative velocity. Find  $v(t)$  if the initial velocity is  $v_0$ .

Answer:

$$v = v_0 e^{\frac{-k}{m}t}$$

5. Solve using Variation of Parameters:  
 $x^2y'' - xy' + y = x^3$

Answer:

$$y = C_1x + C_2x \ln x + \frac{x^3}{4}$$

6. Find the general solution given  $y_1 = x$ :  
 $x^3y'' - xy' + y = 0$

Answer:

$$y = C_1x + C_2xe^{\frac{-1}{x}}$$

7. Solve using Undetermined Coefficients:

$$y''' - y'' = 2 + 2 \cos x$$

Answer:

$$y = C_1 + C_2 x + C_3 e^x - x^2 + \cos x - \sin x$$

8. A 1 kg mass is attached to a spring with spring constant 9 N/m. There is no damping. The mass is initially 2 m below equilibrium position, moving upwards at 1 m/s.

a) Find  $x(t)$

b) Write  $x(t)$  as  $A \sin(\omega t + \phi)$

Answer:

a)  $x = 2 \cos 3t - \frac{1}{3} \sin 3t$

b)  $x = \frac{\sqrt{37}}{3} \sin(3t + 1.74)$

9. Find the first two nonzero terms of  $y_1$  and  $y_2$ :

$$(x + 2)y'' + y = 0$$

Answer:

$$y = C_0(1 - \frac{x^2}{4} + \dots) + C_1(x - \frac{x^3}{12} + \dots)$$

10. Solve using the Laplace transform given  $y(0) = 0, y'(0) = 1$

$$y'' + 4y = \begin{cases} \cos 2t, & 0 \leq t < \pi \\ 0, & t \geq \pi \end{cases}$$

Answer:

$$y(t) = \frac{1}{2} \sin 2t + \frac{1}{4} t \sin 2t - \frac{1}{4} (t - \pi) [\sin 2(t - \pi)] \mathcal{U}(t - \pi)$$

11. Solve  $\vec{X}' = \begin{bmatrix} 2 & 4 \\ 6 & 4 \end{bmatrix} \vec{X}$

Answer:

$$\vec{X} = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-2t} + C_2 \begin{bmatrix} 2 \\ 3 \end{bmatrix} e^{8t}$$