Math 252 – Final Exam Formula Sheet

Trig Integration

$$\int \tan x\,dx = -\ln|\cos x| + C$$
$$= \ln|\sec x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C \quad \text{(textbook)}$$
$$= -\ln|\csc x| + C \quad \text{(Gilles' materials)}$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = \ln|\csc x - \cot x| + C \quad \text{(textbook)}$$
$$= -\ln|\csc x + \cot x| + C \quad \text{(Gilles'materials)}$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

Trigonometry Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2} \qquad \cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \qquad \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Trig Substitution

$$a^2 - x^2 \rightarrow x = a\sin\theta \qquad x^2 + a^2 \rightarrow x = a\tan\theta \qquad x^2 - a^2 \rightarrow x = a\sec\theta$$

Maclaurin Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \qquad \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots \qquad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$$

Integrating Factors

$$e^{\int \frac{M_y - N_x}{N} dx} \qquad e^{\int \frac{N_x - M_y}{M} dy} \qquad \int \frac{e^{-\int P(x)dx}}{y_1^2} dx$$

Misc

$$y = e^{\alpha x}\left(C_1 \cos \beta x + C_2 \sin \beta x\right)$$

$$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix}$$

Systems of DEs

$$\mathbf{X_2} = c_2\left(\mathbf{K_1} t e^{\lambda_1 t} + \mathbf{P} e^{\lambda_1 t}\right)$$

$$\mathbf{X}_1 = \left(\mathbf{B_1} \cos \beta t - \mathbf{B_2} \sin \beta t\right) e^{\alpha t}$$
$$\mathbf{X}_2 = \left(\mathbf{B_1} \sin \beta t + \mathbf{B_2} \cos \beta t\right) e^{\alpha t}$$

$$\mathbf{X_p} = \mathbf{\Phi}(t)\mathbf{U}(t) \qquad \text{where} \qquad \mathbf{U}(t) = \int \mathbf{\Phi}^{-1}(t)\mathbf{F}(t)\,dt$$

Table of Laplace Transforms

$$\mathscr{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt = F(s)$$

$f(t)$	$\mathscr{L}\{f(t)\} = F(s)$	$f(t)$	$\mathscr{L}\{f(t)\} = F(s)$
1	$\frac{1}{s}$	e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$	$\frac{e^{at}t^n}{n!}$	$\frac{1}{(s-a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{at}\sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$	$e^{at}\cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
$t \sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$	$t \cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$\frac{\sin \omega t - \omega t \cos \omega t}{2\omega^3}$	$\frac{1}{(s^2+\omega^2)^2}$	$\mathcal{U}(t-a)$	$\frac{e^{-as}}{s}$
$\delta(t)$	1	$\delta(t-a)$	e^{-as}

$$\mathscr{L}\{\alpha f(t) + \beta g(t)\} = \alpha \mathscr{L}\{f(t)\} + \beta \mathscr{L}\{g(t)\}$$

$$\mathscr{L}\{e^{at} f(t)\} = F(s-a) \qquad \mathscr{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$$

$$\mathscr{L}^{-1}\{e^{-as} F(s)\} = f(t-a)\,\mathcal{U}(t-a) \qquad \mathscr{L}\{f(t)\,\mathcal{U}(t-a)\} = e^{-as} \mathscr{L}\{f(t+a)\}$$

$$\mathscr{L}\{f'(t)\} = sF(s) - f(0) \qquad \mathscr{L}\{f''(t)\} = s^2 F(s) - sf(0) - f'(0)$$

$$\mathscr{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - \cdots - f^{(n-1)}(0)$$

$$f(t) * g(t) = \int_0^t f(\theta) g(t-\theta)\, d\theta \implies \mathscr{L}\{f(t) * g(t)\} = F(s)G(s)$$

$$\mathscr{L}\left\{\int_0^t f(\theta)\, d\theta\right\} = \frac{F(s)}{s}$$

$$f(t) \text{ has period } T \implies \mathscr{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt$$