

Math 252: Formula Sheet for Tests and Final Exam

$$\int \tan x \, dx = \ln |\sec x| + C = -\ln |\cos x| + C \qquad \int \cot x \, dx = -\ln |\csc x| + C = \ln |\sin x| + C$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C \qquad \int \csc x \, dx = -\ln |\csc x + \cot x| + C = \ln |\csc x - \cot x| + C$$

$$\int \sec^2 x \, dx = \tan x + C \qquad \int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C \qquad \int \csc x \cot x \, dx = -\csc x + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\left(\frac{x}{a}\right) + C \qquad \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$$

$$\begin{aligned} \sin^2 \theta + \cos^2 \theta &= 1 & 1 + \tan^2 \theta &= \sec^2 \theta & 1 + \cot^2 \theta &= \csc^2 \theta \\ \cos^2 \theta &= \frac{1 + \cos 2\theta}{2} & \sin^2 \theta &= \frac{1 - \cos 2\theta}{2} & \sin 2\theta &= 2 \sin \theta \cos \theta \end{aligned}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$e^{\int \frac{My - Nx}{N} dx} \quad e^{\int \frac{Nx - My}{M} dy} \quad \int \frac{e^{-\int P(x) dx}}{y_1^2} dx$$

$$y = e^{\alpha x} (c_1 \cos(\beta x) + c_2 \sin(\beta x))$$

$$y = x^\alpha (c_1 \cos(\beta \ln x) + c_2 \sin(\beta \ln x))$$

$$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots, \quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots, \quad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\mathbf{X}(t) = c_1 \mathbf{K} e^{\lambda t} + c_2 (\mathbf{K}t + \mathbf{P}) e^{\lambda t}$$

$$\mathbf{X}(t) = c_1 e^{\alpha t} [\mathbf{B}_1 \cos(\beta t) - \mathbf{B}_2 \sin(\beta t)] + c_2 e^{\alpha t} [\mathbf{B}_2 \cos(\beta t) + \mathbf{B}_1 \sin(\beta t)]$$

$$\mathbf{X}_p(t) = \Phi(t) \int \Phi^{-1}(t) \mathbf{F}(t) dt$$

Table of Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt = F(s)$$

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$	$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
1	$\frac{1}{s}$	e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$	$\frac{e^{at} t^n}{n!}$	$\frac{1}{(s-a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$
$t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
$\frac{\sin \omega t - \omega t \cos \omega t}{2\omega^3}$	$\frac{1}{(s^2 + \omega^2)^2}$	$\mathcal{U}(t-a)$	$\frac{e^{-as}}{s}$
$\delta(t)$	1	$\delta(t-a)$	e^{-as}

$$\mathcal{L}\{\alpha f(t) + \beta g(t)\} = \alpha \mathcal{L}\{f(t)\} + \beta \mathcal{L}\{g(t)\}$$

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a) \quad \mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$$

$$\mathcal{L}^{-1}\{e^{-as} F(s)\} = f(t-a) \mathcal{U}(t-a) \quad \mathcal{L}\{f(t) \mathcal{U}(t-a)\} = e^{-as} \mathcal{L}\{f(t+a)\}$$

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0) \quad \mathcal{L}\{f''(t)\} = s^2 F(s) - sf(0) - f'(0)$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$$

$$f(t) * g(t) = \int_0^t f(\theta) g(t-\theta) d\theta \implies \mathcal{L}\{f(t) * g(t)\} = F(s)G(s)$$

$$\mathcal{L}\left\{\int_0^t f(\theta) d\theta\right\} = \frac{F(s)}{s}$$

$$f(t) \text{ has period } T \implies \mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t) dt$$