1.2 Initial Value Problems (IVP)

Ex: $y' = y$ has solution $y = Ce^x$. Solve:

a) $\left.\begin{array}{l} y' = y \\ y(0) = 3 \end{array}\right\}$ IVP

$y(0) = 3$ ← initial condition

$$\boxed{y = Ce^x}$$

Sub $x = 0$, $y = 3$: $\quad 3 = Ce^0$

$$C = 3$$

$$\boxed{y = 3e^x}$$

b) $\left\{\begin{array}{l} y' = y \\ y(1) = -3 \end{array}\right.$

$$\boxed{y = Ce^x}$$

$$-3 = Ce^1$$

$$C = -3e^{-1}$$

$$y = -3e^{-1}e^x$$

$$\boxed{y = -3e^{x-1}}$$

Ex: $y' + 2xy^2 = 0$ has solution $y = \frac{1}{x^2+C}$

Solve $\begin{cases} y' + 2xy^2 = 0 \\ y(0) = -\frac{1}{4} \end{cases}$

Find interval of solution.

$$\boxed{y = \frac{1}{x^2+C}}$$

Sub $x=0$, $y=-\frac{1}{4}$: $\quad -\frac{1}{4} = \frac{1}{C}$

$$C = -4$$

$$\boxed{y = \frac{1}{x^2-4}}$$

Interval of solution: $x^2 - 4 \neq 0$

$x^2 \neq 4$

$x \neq \pm 2$

Possible intervals:

$x < -2$ or $-2 < x < 2$ or $x > 2$

But interval must contain $x=0$ (initial condition)

$$\boxed{\text{Interval: } -2 < x < 2}$$

Ex: $\frac{d^2x}{dt^2} + 9x = 0$ has solution

$$x = C_1 \cos 3t + C_2 \sin 3t$$

Solve if $x\left(\frac{\pi}{3}\right) = 6$ and $x'\left(\frac{\pi}{3}\right) = 1$

$$\boxed{x = C_1 \cos 3t + C_2 \sin 3t}$$

$t = \frac{\pi}{3}$, $x = 6$:

$$6 = C_1 \cos \pi + C_2 \sin \pi$$

$$6 = -C_1$$

$$C_1 = -6$$

$$\boxed{x = -6 \cos 3t + C_2 \sin 3t}$$

$$x' = 18 \sin 3t + 3C_2 \cos 3t$$

$t = \frac{\pi}{3}$, $x' = 1$:

$$1 = 18 \sin \pi + 3C_2 \cos \pi$$

$$1 = -3C_2$$

$$C_2 = -\frac{1}{3}$$

$$\boxed{x = -6 \cos 3t - \frac{1}{3} \sin 3t}$$