Week 9 Monday

March 2, 2020 7:58 AM

Test FRI MARCH 6
2.3 - 2.4, 3.1 - 3.5
Practice Problems
Videos } www.leahhoward.com

Bring Earplugs / Music

4.1 Eigenvalues and Eigenvectors

Ex: Find a basis for the eigenspace E_8 for $A = \begin{bmatrix} 9 & -2 \\ 5 & -2 \end{bmatrix}$

Solve $[A - \lambda I \mid \vec{0}]$

$[A - 8I \mid \vec{0}]$

$$\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 5 & -10 & 0 \end{array}\right]$$

$$\begin{array}{cc} x_1 & x_2 \\ \end{array}$$
$$\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right] \text{ RREF}$$

$x_2 = t$

$x_1 - 2x_2 = 0 \longrightarrow x_1 = 2t$

eigenvectors $\vec{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} t \quad (t \neq 0)$

basis for $E_8 = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$

FACT

> Let B be an $n \times n$ matrix.
> $B\vec{x} = \vec{0}$ has nontrivial solutions exactly when $\det B = 0$

FACT

> To find eigenvalues of A,
> solve $\det(A - \lambda I) = 0$

Why? λ is an eigenvalue of A

$\iff A\vec{x} = \lambda\vec{x} \quad (\vec{x} \neq \vec{0})$

$\iff (A - \lambda I)\vec{x} = \vec{0} \quad (\vec{x} \neq \vec{0})$

$\iff \det(A - \lambda I) = 0$

Ex: Find all eigenvalues of $A = \begin{bmatrix} 4 & -2 \\ 5 & -7 \end{bmatrix}$

Solve $\det(A - \lambda I) = 0$

$$\begin{vmatrix} 4-\lambda & -2 \\ 5 & -7-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(-7-\lambda) - (-2)(5) = 0$$

$$-28 - 4\lambda + 7\lambda + \lambda^2 + 10 = 0$$

$$\lambda^2 + 3\lambda - 18 = 0$$

$$(\lambda + 6)(\lambda - 3) = 0$$

$$\lambda = -6, 3$$

Summary

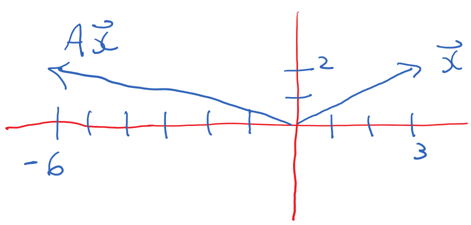
To find eigenvectors : Solve $[A - \lambda I \mid \vec{0}]$ (system)

" eigenvalues : Solve $\det(A - \lambda I) = 0$ (equation)

Ex: $A = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$

Find the eigenvectors and eigenvalues geometrically.

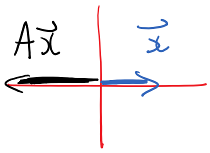
$$A\vec{x} = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2x_1 \\ x_2 \end{bmatrix}$$



For which $\vec{x}$ are $\vec{x}$ and $A\vec{x}$ parallel?

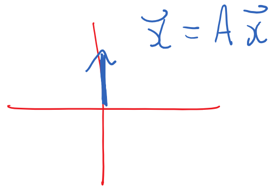
x-axis

x-axis



$A\vec{x} = -2\vec{x}$ $\quad \lambda = -2$

y-axis



$\lambda = 1$

$$E_{-2} = \text{span}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$$
$$E_1 = \text{span}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

4.2 Determinants

(Return to eigenvalues in 4.3)

Notation: $\det A$ or $|A|$

$\det A$ is only defined if A is $n \times n$

Cofactor Expansion

- Can expand along any row or column

$$\begin{bmatrix} + & - & \\ - & + & \\ + & & \ddots \end{bmatrix}$$ checkerboard pattern

Ex: $A = \begin{bmatrix} 4 & 1 & 6 \\ 1 & 2 & 3 \\ 6 & 0 & 7 \end{bmatrix}$

Find $\det A$

3rd row: $|A| = 6\begin{vmatrix} 1 & 6 \\ 2 & 3 \end{vmatrix} - 0\begin{vmatrix} 4 & 6 \\ 1 & 3 \end{vmatrix} + 7\begin{vmatrix} 4 & 1 \\ 1 & 2 \end{vmatrix}$ $\quad \begin{bmatrix} & & \\ & & \\ + & - & + \end{bmatrix}$

$$= 6(-9) + 7(7)$$
$$= -5$$

Alternatively:

2^{nd} column: $|A| = -1\begin{vmatrix} 1 & 3 \\ 6 & 7 \end{vmatrix} + 2\begin{vmatrix} 4 & 6 \\ 6 & 7 \end{vmatrix} \cancel{- 0\begin{vmatrix} & \\ & \end{vmatrix}}$ $\quad \begin{bmatrix} & - & \\ & + & \\ & - & \end{bmatrix}$

$$= -1(-11) + 2(-8)$$
$$= -5$$

etc.

Ex: $\begin{vmatrix} 1 & 6 & 2 & 3 \\ 0 & 0 & 0 & 4 \\ 2 & 1 & 1 & 6 \\ 2 & 0 & 5 & 7 \end{vmatrix}$ $\quad \begin{bmatrix} + & & & \\ - & + & - & + \\ & & & \end{bmatrix}$

$$= 4\begin{vmatrix} 1 & 6 & 2 \\ 2 & 1 & 1 \\ 2 & 0 & 5 \end{vmatrix} \qquad \begin{bmatrix} + & & \\ - & & \\ + & - & + \end{bmatrix}$$

$$= 4\left[2\begin{vmatrix} 6 & 2 \\ 1 & 1 \end{vmatrix} + 5\begin{vmatrix} 1 & 6 \\ 2 & 1 \end{vmatrix} \right]$$

$$= 4\left[2(4) + 5(-11) \right]$$

$$= -188$$