Week 8 Wednesday

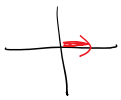
February 26, 2020 8:54 AM

3.6 Linear Transformations Cont'd

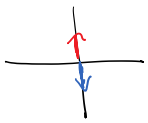
Ex: $T: \mathbb{R}^2 \to \mathbb{R}^2$

Reflects a vector in x-axis

Find $[T]$



$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix}$$



$$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}0\\-1\end{bmatrix}$$

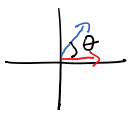
$$[T] = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$$

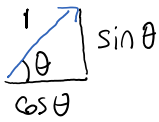
$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) \quad T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$$

Ex: $T: \mathbb{R}^2 \to \mathbb{R}^2$

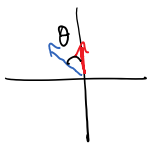
Rotates a vector by angle θ (counterclockwise)

Find $[T]$





$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}\cos\theta\\\sin\theta\end{bmatrix}$$



$$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}-\sin\theta\\\cos\theta\end{bmatrix}$$

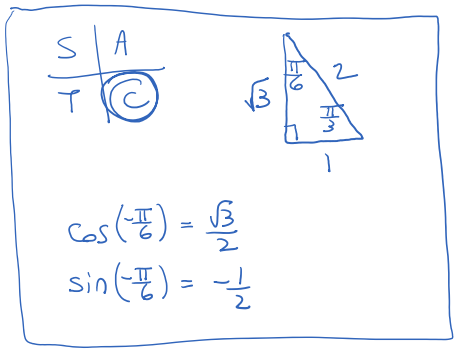
$$[T] = \begin{bmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{bmatrix}$$

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) \quad T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$$

 Know this

Ex: Rotate $\begin{bmatrix}1\\1\end{bmatrix}$ by 30° clockwise

$$[T] = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}_{\theta=-30^\circ}$$



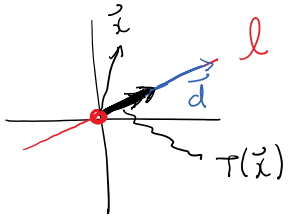
$$= \frac{1}{2}\begin{bmatrix}\sqrt{3} & 1\\ -1 & \sqrt{3}\end{bmatrix}$$

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix}\sqrt{3} & 1\\ -1 & \sqrt{3}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}\sqrt{3}+1\\ -1+\sqrt{3}\end{bmatrix}$$

Ex: $T: \mathbb{R}^2 \to \mathbb{R}^2$ projects a vector on the line through the origin with $\vec{d} = \begin{bmatrix}a\\b\end{bmatrix}$

Find $[T]$



$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \text{proj}_{\vec{d}}\begin{bmatrix}1\\0\end{bmatrix} \qquad \vec{d} = \begin{bmatrix}a\\b\end{bmatrix}$$

$$= \frac{\vec{d}\cdot\begin{bmatrix}1\\0\end{bmatrix}}{\|\vec{d}\|^2}\vec{d}$$

$$= \frac{a}{a^2+b^2}\begin{bmatrix}a\\b\end{bmatrix}$$

$$= \frac{1}{a^2+b^2}\begin{bmatrix}a^2\\ab\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \text{proj}_{\vec{d}}\begin{bmatrix}0\\1\end{bmatrix} \qquad \vec{d} = \begin{bmatrix}a\\b\end{bmatrix}$$

$$= \frac{\vec{d} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{\| \vec{d} \|^2} \vec{d}$$

$$= \frac{b}{a^2+b^2} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$= \frac{1}{a^2+b^2} \begin{bmatrix} ab \\ b^2 \end{bmatrix}$$

$$[T] = \frac{1}{a^2+b^2} \begin{bmatrix} a^2 & ab \\ ab & b^2 \end{bmatrix}$$ (★) Know this

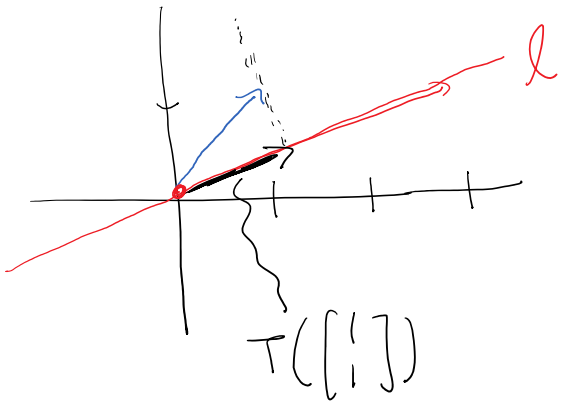
Ex: Project $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ onto the line through the origin with $\vec{d} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$$[T] = \frac{1}{a^2+b^2} \begin{bmatrix} a^2 & ab \\ ab & b^2 \end{bmatrix}_{\substack{a=3 \\ b=1}}$$

$$= \frac{1}{10} \begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \frac{1}{10} \begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 12 \\ 4 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 6/5 \\ 2/5 \end{bmatrix}$$



Suppose we apply T_1 then T_2 to $\vec{x}$

Notation: $T_2(T_1(\vec{x}))$ or $(T_2 \circ T_1)(\vec{x})$

Calculation: $[T_2][T_1]\vec{x}$