

Test 2 FRI MARCH 6

2.3-2.4, 3.1-3.5 (6 Questions)

Practice Problems on website

Final Exam Thurs April 16th
1:30pm CC124

3.5 Subspaces and Dimension Cont'd

Recap of $\text{row}(A)$, $\text{col}(A)$, $\text{null}(A)$:
Week 6 Friday notes

Def

The dimension of a subspace is the number of basis vectors

Ex: a) Standard basis for $\mathbb{R}^3 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

$$\dim(\mathbb{R}^3) = 3$$

b) $\dim(\mathbb{R}^n) = n$

c) $\dim(\text{plane}) = 2$

d) $\dim(\text{line}) = 1$

Def

The rank of a matrix is the # of leading entries in REF/RREF

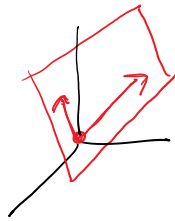
Rephrased: $\text{rank} = \dim[\text{row}(A)] = \dim[\text{col}(A)]$

Def

The nullity of a matrix is the # of parameters in solution to $A\vec{x} = \vec{0}$

Rephrased: nullity = $\dim[\text{null}(A)]$

Quick Ex:



null(A)

$$\text{nullity}(A) = 2$$

Ex: $A = \begin{bmatrix} 1 & 5 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$
Find the rank and nullity

$\rightsquigarrow \begin{bmatrix} \textcircled{1} & 5 & 0 & 0 \\ 0 & 0 & \textcircled{1} & 0 \\ 0 & 0 & 0 & \textcircled{1} \end{bmatrix}$ RREF
rank = 3

nullity: $\begin{bmatrix} \textcircled{1} & 5 & 0 & 0 & | & 0 \\ 0 & 0 & \textcircled{1} & 0 & | & 0 \\ 0 & 0 & 0 & \textcircled{1} & | & 0 \end{bmatrix}$
↑
nullity = 1

Fact

For any matrix A,
rank + nullity = # columns in A

Intuitively:

$\boxed{\text{\# Columns with circles}} + \boxed{\text{\# Columns without circles}} = \text{\# Columns}$

Summary

Subspaces Associated with A

Name for the Dim. of Subspace

How to Find Dim. of Subspace

row space

rank

leading entries in REF/RREF

column space

rank

null space

nullity

#columns - rank

Handout

"Long Fundamental Theorem"

For any given matrix,

all 15 statements are true

or

"

false

a-e

Section 3.3

f, g

h-j

Inf about columns of A
" rows "

k-m

n, o

Ch 4

Ex: Is $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix} \right\}$
a basis for \mathbb{R}^3 ?

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 6 \\ 1 & 4 & 4 \end{bmatrix}$$

m) is true exactly when f) is true

If $\text{rank}(A) = 3$, yes
If $\text{rank}(A) \neq 3$, no

$$A \rightsquigarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \text{ REF}$$

$$\text{rank}(A) = 3$$

Yes