Week 6 Friday

February 14, 2020 9:56 AM

3.5 Subspaces and Dimension Cont'd

Ch 7: Best-fit curve through a set of points?

Need column space and null space

Ex: $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 0 & 0 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 0 \\ 1 & 2 & 0 \end{bmatrix}$

RREF of $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ RREF of $A^T = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Find a basis for:

a) row(A)

Use nonzero rows of REF/RREF

$\{ [1 \ 0 \ 0], [0 \ 1 \ 1] \}$

b) col(A)

Use leading entries of REF/RREF as a guide

$\begin{bmatrix} 1 & & \\ & 1 & \\ & & \end{bmatrix}$

Use Columns 1 and 2 <u>of A</u>

$\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$

- Row ops change column space (can't use REF/RREF)
- Columns don't move around during row ops

c) row(A), consisting of rows of A

$\text{row}(A) = \text{col}(A^T)$

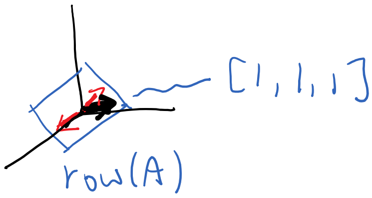
Find a basis for $\text{col}(A^T)$

RREF of $A^T = \begin{bmatrix} 1 & & \\ & & 1 \end{bmatrix}$

Use columns 1 and 3 of A^T

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

or $\{ [1 \ 1 \ 1], [1 \ 0 \ 0] \}$



Ex: $A = \begin{bmatrix} 1 & 4 & 6 \\ 2 & 8 & 12 \end{bmatrix}$

Find a basis for null(A)

Solutions to $A\vec{x} = \vec{0}$

$$\left[\begin{array}{ccc|c} 1 & 4 & 6 & 0 \\ 2 & 8 & 12 & 0 \end{array}\right]$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \end{array}$$

$R_2 - 2R_1$ $\left[\begin{array}{ccc|c} 1 & 4 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ RREF

$\boxed{x_2 = s}$ $\boxed{x_3 = t}$

$x_1 + 4x_2 + 6x_3 = 0$ $\boxed{x_1 = -4s - 6t}$

$$\vec{x} = \begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -6 \\ 0 \\ 1 \end{bmatrix} t$$

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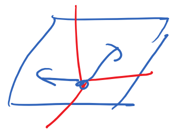
Each parameter gives a basis vector

Basis for null(A) $= \left\{ \begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 0 \\ 1 \end{bmatrix} \right\}$

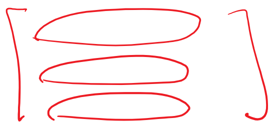
null(A)



basis $= \{ \vec{d}_1, \vec{d}_2 \}$

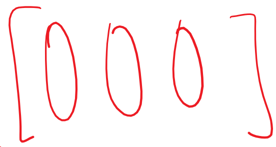
<u>Ex</u>: Find a basis for $\operatorname{span}\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 24 \end{bmatrix} \right)$

<u>Method I</u>



Find a basis for row(A)

<u>Method II</u>



Find a basis for col(A)

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 6 \\ 1 & 5 & 24 \end{bmatrix}$$

Find a basis for row(A)

$$\rightsquigarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{REF}$$

Basis $= \{ [1 \;\; 1 \;\; 0], [0 \;\; 1 \;\; 6] \}$

Def

Given a basis $\mathcal{B} = \{ \vec{v}_1, \vec{v}_2, \dots, \vec{v}_n \}$ for $\mathbb{R}^n$, the <u>coordinate vector of $\vec{v}$ with respect to $\mathcal{B}$</u> is $[\vec{v}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$ such that $\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_n\vec{v}_n$

Ex: Find $[\vec{v}]_{\mathcal{B}}$ for $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \right\}$

and $\vec{v} = \begin{bmatrix} 5 \\ 15 \\ 28 \end{bmatrix}$

Solve $c_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \\ 28 \end{bmatrix}$

$$\begin{array}{ccc} c_1 & c_2 & c_3 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 2 & 5 & 1 & 15 \\ 3 & 6 & 4 & 28 \end{array}\right]$$

$$\begin{array}{ccc} c_1 & c_2 & c_3 \end{array}$$
$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$c_1 = -2$

$c_2 = 3$

$c_3 = 4$

$$[\vec{v}]_{\mathcal{B}} = \begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix}$$