Week 6 Thursday

February 13, 2020 9:40 AM

3.5 Subspaces and Dimension Cont'd

FACT

$\boxed{\text{A set of vectors } S \text{ is a subspace of } \mathbb{R}^n \text{ if and only if } S \text{ is the span of some vector(s)}}$

e.g. line through origin
plane through origin
etc.

3 Subspaces Associated with a Matrix

- row space of A : span of the rows of A
- column space of A : " columns "
- null space of A : $\{ \vec{x} \text{ such that } A\vec{x} = \vec{0} \}$

Notation: row(A), col(A), null(A)

Ex: $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

a) Is $\begin{bmatrix} 6 \\ 10 \end{bmatrix}$ in col(A) ?

$$c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 2 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \end{bmatrix} ?$$

$$6 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 2 \end{bmatrix} + 4 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

$\boxed{\text{Yes}}$

b) Is $[1 \ 2 \ 5]$ in row(A) ?

$$c_1 [1 \ 2 \ 0] + c_2 [1 \ 2 \ 1] = [1 \ 2 \ 5] \ ?$$

$$\begin{array}{cc|c} c_1 & c_2 & \\ \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 0 & 1 & 5 \end{array}\right]$$
$$\begin{array}{cc|c} c_1 & c_2 & \\ \end{array}$$
$$\rightsquigarrow \left[\begin{array}{cc|c} 1 & 0 & -4 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{array}\right] \text{ RREF}$$

$c_1 = -4$
$c_2 = 5$

$\boxed{\text{Yes}}$

To check: $-4[1 \ 2 \ 0] + 5[1 \ 2 \ 1] = [1 \ 2 \ 5]$ ✓

c) Is $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in null(A) ?

$$A\vec{x} = \vec{0} \ ?$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \neq \vec{0} \qquad \boxed{\text{No}}$$

Def

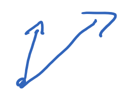
A set $\mathcal{B}$ is a <u>basis</u> for a subspace S if:

1) span$(\mathcal{B}) = S$
2) $\mathcal{B}$ is linearly independent

$\vec{v}_1, \vec{v}_2$ are lin. independent if

$$c_1\vec{v}_1 + c_2\vec{v}_2 = \vec{0}$$

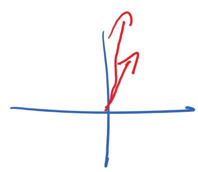
$$\Rightarrow \ 0\vec{v}_1 + 0\vec{v}_2 = \vec{0}$$



Intuition: A basis contains enough vectors to describe S, and has no redundancy.

Ex: a) $\left\{ \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix} \right\}$ is a basis for $\mathbb{R}^2$

b) $\left\{ \begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix} \right\}$ "



c) $\left\{ \begin{bmatrix}1\\2\end{bmatrix} \right\}$ is not a basis for $\mathbb{R}^2$
(Property 1 fails)

d) $\left\{ \begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix}, \begin{bmatrix}3\\6\end{bmatrix} \right\}$ is not a basis for $\mathbb{R}^2$
(not lin. ind.)

Ex: $A = \begin{bmatrix} 2 & 3 & 7 \\ 4 & 7 & 10 \\ 8 & 17 & 8 \end{bmatrix}$

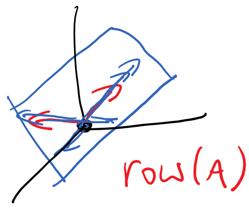
Find a basis for:

a) row(A)

USE NONZERO ROWS OF REF / RREF

$$\rightsquigarrow \begin{bmatrix} 2 & 3 & 7 \\ 0 & 1 & -4 \\ 0 & 0 & 0 \end{bmatrix} \ \text{REF}$$

Basis for row(A) = { [2 3 7], [0 1 -4] }



b) col(A)

$\boxed{\text{USE COLUMNS OF A CORRESPONDING TO LEADING ENTRIES OF REF/RREF}}$

$$\begin{bmatrix} 2 & 3 & 7 \\ 0 & 1 & -4 \\ 0 & 0 & 0 \end{bmatrix} \text{ REF}$$

Use Columns 1 and 2 of A

Basis for col(A) $= \left\{ \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ 17 \end{bmatrix} \right\}$

Caution: Can't use Columns from REF/RREF

Row operations change the column space

$\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ is not in col(A)

c) row(A), consisting of rows of A

Different than a)

[0 1 -4] is not a row of A

Rows of A = Columns of A^T

→ Find a basis for col(A^T)

$$A^T = \begin{bmatrix} 2 & 4 & 8 \\ 3 & 7 & 17 \\ 7 & 10 & 8 \end{bmatrix}$$

'' $\begin{bmatrix} 7 & 10 & 8 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{REF}$$

Use Columns 1 and 2 of A^T

$$\text{Basis} = \left\{ \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}, \begin{bmatrix} 4 \\ 7 \\ 10 \end{bmatrix} \right\}$$

$$\text{or } \left\{ \begin{bmatrix} 2 & 3 & 7 \end{bmatrix}, \begin{bmatrix} 4 & 7 & 10 \end{bmatrix} \right\}$$