Week 6 Wednesday

February 12, 2020 9:03 AM

3.4 LU Factorization Cont'd

Ex: Find LU and use it to solve

$$\begin{bmatrix} 2 & -4 & 0 \\ 3 & -1 & 4 \\ -1 & 2 & 2 \end{bmatrix} \vec{x} = \begin{bmatrix} 2 \\ 0 \\ -5 \end{bmatrix}$$

1. Find LU

$$\begin{matrix} R_2 - \frac{3}{2}R_1 \\ R_3 + \frac{1}{2}R_1 \end{matrix} \quad \begin{bmatrix} 2 & -4 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 2 \end{bmatrix} \quad \begin{matrix} k = \frac{3}{2} \\ k = -\frac{1}{2} \end{matrix}$$

$$\boxed{\begin{matrix} 3 + ?(2) = 0 \\ ? = -\frac{3}{2} \end{matrix}}$$

$$R_3 - 0R_2 \quad \begin{bmatrix} 2 & -4 & 0 \\ 0 & 5 & 4 \\ 0 & \underline{\underline{0}} & 2 \end{bmatrix} \quad k = 0$$

REF

$U = REF$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{bmatrix}$$

k-values

$A = LU$

2.

$$A\vec{x} = \vec{b}$$

$$L\underbrace{U\vec{x}}_{\vec{y}} = \vec{b}$$

Solve $L\vec{y} = \vec{b}$

$$\begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ \frac{3}{2} & 1 & 0 & 0 \\ \frac{-1}{2} & 0 & 1 & -5 \end{array}\right] \end{array}$$

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$\boxed{y_1 = 2}$

$\underbrace{\cancel{\tfrac{3}{2}y_1}}_{3} + y_2 = 0 \rightarrow \boxed{y_2 = -3}$

$\underbrace{\cancel{-\tfrac{1}{2}y_1}}_{-1} + y_3 = -5 \rightarrow \boxed{y_3 = -4}$

$$\vec{y} = \begin{bmatrix} 2 \\ -3 \\ -4 \end{bmatrix}$$

3 Solve $U\vec{x} = \vec{y}$

$$\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 2 & -4 & 0 & 2 \\ 0 & 5 & 4 & -3 \\ 0 & 0 & 2 & -4 \end{array}\right] \end{array}$$

Start @ bottom

$2x_3 = -4 \rightarrow \boxed{x_3 = -2}$

$5x_2 + \underbrace{\cancel{4x_3}}_{-8} = -3 \rightarrow \boxed{x_2 = 1}$

$2x_1 \underbrace{\cancel{-4x_2}}_{-4} = 2 \rightarrow \boxed{x_1 = 3}$

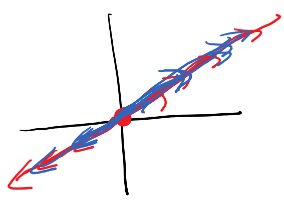
$$\vec{x} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$$

3.5 Subspaces and Dimension

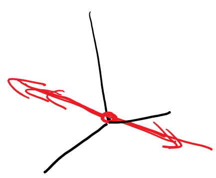
Def

> A **subspace** of $\mathbb{R}^n$ is a collection of vectors S in $\mathbb{R}^n$ such that:
>
> 1) $\vec{0}$ is in S
> 2) If $\vec{u}$ is in S then $c\vec{u}$ is in S (c: any real #)
> 3) If $\vec{u}$ and $\vec{v}$ are in S then $\vec{u}+\vec{v}$ is in S

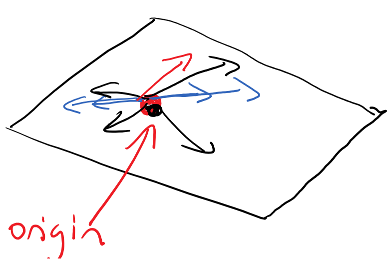
Ex: A line through the origin in $\mathbb{R}^2$ is a subspace of $\mathbb{R}^2$

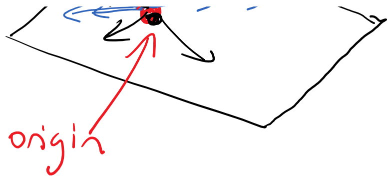


Ex: " $\mathbb{R}^3$ " $\mathbb{R}^3$



Ex: A plane through origin in $\mathbb{R}^3$ is a subspace of $\mathbb{R}^3$





Ex: Subspace of $\mathbb{R}^3$?

$$S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \ x = z+1 \right\}$$

$$\boxed{\begin{array}{l} \text{Yes if all 3 properties are true} \\ \text{No if } \geq 1 \text{ property is false} \end{array}}$$

$$S = \left\{ \begin{bmatrix} z+1 \\ y \\ z \end{bmatrix} \right\}$$

No because $\vec{0}$ is not in S.

Ex: Subspace of $\mathbb{R}^2$?

$$S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \ y = x^3 \right\}$$

$$S = \left\{ \begin{bmatrix} x \\ x^3 \end{bmatrix} \right\}$$

$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is in S

$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ is not in S

Property 2 fails.

$\boxed{\text{No}}$

To show that a property fails, can use numbers.

" " " " " is true, must use variables.

Ex: Show that $S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \; 3x - 4y = 0 \right\}$ is a subspace of $\mathbb{R}^2$.

$$3x - 4y = 0$$
$$3x = 4y$$
$$x = \frac{4}{3}y$$
$$S = \left\{ \begin{bmatrix} \frac{4}{3}y \\ y \end{bmatrix} \right\}$$

1) Let $y = 0$: $\vec{0}$ is in S ✓

2) Let $\vec{u} = \begin{bmatrix} \frac{4}{3}a \\ a \end{bmatrix}$

$c\vec{u} = \begin{bmatrix} \frac{4}{3}ca \\ ca \end{bmatrix}$ is in S (correct form)

3) Let $\vec{u} = \begin{bmatrix} \frac{4}{3}a \\ a \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} \frac{4}{3}b \\ b \end{bmatrix}$

$$\vec{u} + \vec{v} = \begin{bmatrix} \frac{4}{3}a + \frac{4}{3}b \\ a+b \end{bmatrix}$$

$= \begin{bmatrix} \frac{4}{3}(a+b) \\ a+b \end{bmatrix}$ is in S (correct form)