Week 6 Tuesday

February 11, 2020 8:47 AM

Test 2 Fri Mar 6

$\boxed{2.3-2.4, \quad 3.1-3.5}$

3.3 The Inverse of a Matrix Cont'd

Handout : "Short Fundamental Theorem"

For any given nxn matrix,
all 5 statements are true OR
" false

Ex: $A = \begin{bmatrix} 1 & 4 \\ 6 & 9 \end{bmatrix}$

A^{-1} exists (det $A \neq 0$)

a)
b)
c)
d)
e)

} all true

Ex: $B = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

B^{-1} is undefined (det $B = 0$)

a)
b)
c)
d)
e)

} all false

d)
e)

3.4 LU Factorization

An <u>upper triangular matrix</u> has 0's below diagonal

$$\begin{bmatrix} 1 & 0 & 9 \\ 0 & 3 & -2 \\ 0 & 0 & -4 \end{bmatrix}$$

A <u>lower triangular matrix</u> " <u>above</u> "

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 0 & -3 & 0 \end{bmatrix}$$

A <u>unit lower triangular matrix</u> is lower triangular and has 1's on diagonal.

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

$$\boxed{A = \underbrace{L}_{\substack{\text{unit} \\ \text{lower} \\ \triangle}} \underbrace{U}_{\substack{\text{upper} \\ \triangle}} \qquad \text{"LU Factorization"}}$$

e.g.

$$\begin{bmatrix} 2 & 1 & 1 \\ 4 & 4 & 3 \\ 8 & 10 & 13 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 6 \end{bmatrix}$$

Ex: Solve $\left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 4 & 4 & 3 & 2 \\ 8 & 10 & 13 & -8 \end{array}\right]$

using the LU factorization above

(1) $A\vec{x} = \vec{b}$

$$L\underbrace{U\vec{x}}_{\vec{y}} = \vec{b}$$

Solve $L\vec{y} = \vec{b}$: $\begin{array}{ccc} y_1 & y_2 & y_3 \end{array}$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 2 \\ 4 & 3 & 1 & -8 \end{array}\right]$

Start @ top

$y_1 = 1$

$2\cancel{y_1}^{2} + y_2 = 2 \rightarrow \quad y_2 = 0$

$4\cancel{y_1}^{4} + \cancel{3y_2} + y_3 = -8 \rightarrow \quad y_3 = -12$

$$\vec{y} = \begin{bmatrix} 1 \\ 0 \\ -12 \end{bmatrix}$$

(2) Solve $U\vec{x} = \vec{y}$

$\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}$
$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 6 & -12 \end{array}\right]$

Start @ bottom

$6x_3 = -12 \rightarrow \quad x_3 = -2$

$2x_2 + \cancel{x_3}^{-2} = 0 \rightarrow \quad x_2 = 1$

$$2x_1 + \cancel{x_2} + \cancel{x_3} = 1 \rightarrow \quad x_1 = 1$$

$$\quad\quad\ \ 1 \quad\ \ -2$$

$$\vec{x} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

Finding LU

$A \rightsquigarrow REF$ using <u>only</u> (current row) $-$ k (pivot row)

Record the k-value

(Only possible when no row swaps are required)

<u>Ex</u>: Find LU for $\begin{bmatrix} 2 & 1 & 1 \\ 4 & 4 & 3 \\ 8 & 10 & 13 \end{bmatrix}$

$$\begin{matrix} R_2 - 2R_1 \\ R_3 - 4R_1 \end{matrix} \quad \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & \underline{\underline{6}} & 9 \end{bmatrix} \begin{matrix} k=2 \\ k=4 \end{matrix}$$

$$R_3 - 3R_2 \quad \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 6 \end{bmatrix} k=3$$

$U = REF$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & \underline{\underline{3}} & 1 \end{bmatrix}$$

k-values

Why this Works

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Elementary matrix for $R_2 \to R_2 - 2R_1$: $E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

L <u>undoes</u> the operations that turn A into U

L turns U into A

$$LU = A$$