3.2 Matrix Algebra Cont'd

Def
A and B <u>commute</u> if $AB = BA$

<u>Ex</u>: Do A and B commute?

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 9 \\ 3 & 7 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 9 \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 11 & 39 \\ 13 & 37 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 9 \\ 3 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 39 \\ 13 & 37 \end{bmatrix}$$

Yes

6 Properties of Matrices

① $(AB)C = A(BC)$

<u>Ex</u>: Confirm for $A = [1 \ \ 3]$ $B = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$ $C = [1 \ \ 6]$

$$(AB)C = [-10][1 \ \ 6] = [-10 \ \ -60]$$

$$A(BC) = [1 \ \ 3]\begin{bmatrix} 2 & 12 \\ -4 & -24 \end{bmatrix} = [-10 \ \ -60] \checkmark$$

② $A(B+C) = AB + AC$

③ $AI = A$
④ $IA = A$

Section 3.1

Ex: $\begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \end{bmatrix}$

⑤ $(A \pm B)^T = A^T \pm B^T$

means $\begin{cases} (A+B)^T = A^T + B^T \\ (A-B)^T = A^T - B^T \end{cases}$

Ex: Confirm 1st formula for $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 4 \\ 1 & 2 \end{bmatrix}$

$(A+B)^T = \begin{bmatrix} 2 & 6 \\ 2 & 5 \end{bmatrix}^T = \begin{bmatrix} 2 & 2 \\ 6 & 5 \end{bmatrix}$

$A^T + B^T = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 6 & 5 \end{bmatrix}$ ✓

⑥ $(A_1 A_2 \cdots A_n)^T = A_n^T \cdots A_2^T A_1^T$ ✪

Ex: Check for $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 4 \\ 1 & 2 \end{bmatrix}$

$(AB)^T = \begin{bmatrix} 3 & 8 \\ 4 & 10 \end{bmatrix}^T = \begin{bmatrix} 3 & 4 \\ 8 & 10 \end{bmatrix}$

$B^T A^T = \begin{bmatrix} 1 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 8 & 10 \end{bmatrix}$ ✓

Ex: Expand $(A+B)^2$ and simplify

$$\begin{aligned} (A+B)^2 &= (A+B)(A+B) \\ &= AA + AB + BA + BB \\ &= A^2 + AB + BA + B^2 \end{aligned}$$

Ex: Show that $A^T A$ is symmetric.

M is symmetric if $M^T = M$

Show that $(A^T A)^T = A^T A$

$$(A^T A)^T = A^T (A^T)^T$$
$$= A^T A \checkmark$$

3.3 The Inverse of a Matrix

Def
An $n \times n$ matrix A is <u>invertible</u> if there exists a matrix A^{-1} (also $n \times n$) so that $AA^{-1} = I$ and $A^{-1}A = I$

A^{-1} is called the <u>inverse</u> of A

<u>Ex</u>: Confirm that $\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$ is the inverse of $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

$$\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} = I \checkmark$$

<u>Comments</u>: 1) Not every square matrix is invertible.

Invertible $\Rightarrow$ Square

Square $\not\Rightarrow$ Invertible

2) Only need to check 1 of

$AA^{-1} = I$ or $A^{-1}A = I$