Week 5 Wednesday

February 5, 2020 9:05 AM

3.2 Matrix Algebra

Ex: Is $\begin{bmatrix} 0 & 1 \\ 4 & 2 \end{bmatrix}$ a linear combination of $\begin{bmatrix} 0 & 1 \\ 6 & 2 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 7 & 2 \end{bmatrix}$?

Let $\underbrace{c_1 \begin{bmatrix} 0 & 1 \\ 6 & 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 1 \\ 7 & 2 \end{bmatrix}}_{\text{lin. com.}} = \begin{bmatrix} 0 & 1 \\ 4 & 2 \end{bmatrix}$

$$\begin{array}{ccc} c_1 & c_2 & \\ \end{array}$$
$$\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 6 & 7 & 4 \\ 2 & 2 & 2 \end{array}\right]$$

Reorder rows $\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 6 & 7 & 4 \\ 2 & 2 & 2 \\ 0 & 0 & 0 \end{array}\right]$

$R_2 - 6R_1$
$R_3 - 2R_1$

$$\begin{array}{ccc} c_1 & c_2 & \\ \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \quad \text{REF}$$

System is consistent

$\boxed{\text{Yes}}$

To check: $c_2 = -2$
$c_1 = 3$

$$3 \begin{bmatrix} 0 & 1 \\ 6 & 2 \end{bmatrix} - 2 \begin{bmatrix} 0 & 1 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & 2 \end{bmatrix} \checkmark$$

Ex: Find the general form of

$$\text{span}\left(\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 6 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} \right)$$

Let $c_1 \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 3 & 0 \\ 6 & 1 \end{bmatrix} + c_3 \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$

→ Want conditions on w, x, y, z

Each zero row of REF gives a condition.

$$\begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & w \\ 0 & 0 & 1 & x \\ 2 & 6 & 4 & y \\ 0 & 1 & 5 & z \end{array}\right]$$

$R_3 - 2R_1$ $\left[\begin{array}{ccc|c} 1 & 3 & 2 & w \\ 0 & 0 & 1 & x \\ 0 & 0 & 0 & y-2w \\ 0 & 1 & 5 & z \end{array}\right]$

Reorder rows $\left[\begin{array}{ccc|c} 1 & 3 & 2 & w \\ 0 & 1 & 5 & z \\ 0 & 0 & 1 & x \\ 0 & 0 & 0 & y-2w \end{array}\right]$ REF ✓

Consistent system

$\Longrightarrow \quad y - 2w = 0$

General form $= \left\{ \begin{bmatrix} w & x \\ y & z \end{bmatrix} \text{ such that } \cancel{y-2w=0} \; y = 2w \right\}$

$= \left\{ \begin{bmatrix} w & x \\ 2w & z \end{bmatrix} \right\}$

Follow Up: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is not in the span

$\begin{bmatrix} -11 & \pi \\ -22 & \sqrt{2} \end{bmatrix}$ <u>is</u> in the span

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<u>Ex</u>: Find the general form of

$$\text{span}\left(\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} \right)$$

Let $c_1 \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$

$$\begin{array}{cc} c_1 & c_2 \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 2 & w \\ 0 & 1 & x \\ 2 & 4 & y \\ 0 & 5 & z \end{array}\right]$$

$R_3 - 2R_1$ $\left[\begin{array}{cc|c} 1 & 2 & w \\ 0 & 1 & x \\ 0 & 0 & y-2w \\ 0 & 5 & z \end{array}\right]$

$R_4 - 5R_2$ $\left[\begin{array}{cc|c} 1 & 2 & w \\ 0 & 1 & x \\ 0 & 0 & y-2w \\ 0 & 0 & z-5x \end{array}\right]$ REF

Consistent system

$\Rightarrow$ $\cancel{y-2w}=0$ and $\cancel{z-5x}=0$

$y=2w$ $\qquad$ $z=5x$

General Form $= \left\{ \begin{bmatrix} w & x \\ 2w & 5x \end{bmatrix} \right\}$

$\begin{bmatrix} 5 & 10 \\ 10 & 50 \end{bmatrix}$ is in the span

$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ is <u>not</u> in span

<u>Ex</u>: Are $\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 6 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$ linearly independent ?

$$c_1 \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} + c_2 \begin{bmatrix} & \\ & \end{bmatrix} + c_3 \begin{bmatrix} & \\ & \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 6 & 4 & 0 \\ 0 & 1 & 5 & 0 \end{array}\right]$$

1 solution $(c_1 = c_2 = c_3 = 0)$ $\rightarrow$ <u>linearly independent</u>

∞-many solutions $\rightarrow$ <u>linearly dependent</u>

$$\rightsquigarrow \begin{array}{ccc} c_1 & c_2 & c_3 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \text{REF}$$

1 solution

$\boxed{\text{Yes}}$

~~$c_3 = 0$~~
~~$c_2 + 5c_3 = 0 \rightarrow c_2 = 0$~~
~~$c_1 + 3c_2 + 2c_3 = 0 \rightarrow c_1 = 0$~~
Overkill