Week 5 Monday

February 3, 2020 7:56 AM

3.1 Matrix Operations

Size of a matrix : (# rows) x (# columns)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad \text{is} \quad 2 \times 3$$

Entry of a matrix : $a_{23} = 6$ or $[A]_{21} = 4$

Square matrix : 2×2, 3×3 etc.

Diagonal matrix : $D = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ etc.

Identity matrix : $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Ex: $A = \begin{bmatrix} 1 & 6 & 1 \\ -2 & -2 & 4 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & 0 & -3 \\ 1 & 6 & 9 \end{bmatrix}$

Find :

a) $A+B$

$$= \begin{bmatrix} 2 & 6 & -2 \\ -1 & 4 & 13 \end{bmatrix}$$

$A+B$ is undefined if A, B have different sizes

b) $-4B$ "scalar multiplication"

$$= \begin{bmatrix} -4 & 0 & 12 \\ -4 & -24 & -36 \end{bmatrix}$$

c) $A - 3B$

$$= A + (-3B)$$

$$= \begin{bmatrix} 1 & 6 & 1 \\ -2 & -2 & 4 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 9 \\ -3 & -18 & -27 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 6 & 10 \\ -5 & -20 & -23 \end{bmatrix}$$

Def

- The <u>transpose</u> of A, written A^T, has rows and columns interchanged.
- A is <u>symmetric</u> if $A^T = A$

<u>Ex</u>: a) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

A is not symmetric ($A^T \neq A$)

b) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

A is symmetric ($A^T = A$)

Matrix Multiplication

$$AB = \begin{bmatrix} r_1 \cdot c_1 & r_1 \cdot c_2 & \\ r_2 \cdot c_1 & \text{etc.} & \end{bmatrix}$$

where r_i = i^{th} row of A

c_j = j^{th} column of B

Ex:

$$\begin{bmatrix} 1 & 4 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 9 & 27 \\ -2 & 0 & 0 \end{bmatrix}$$

$\boxed{\begin{bmatrix} 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}}$

$\boxed{\begin{bmatrix} 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$

$\boxed{\begin{bmatrix} -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}}$

Consider the sizes:

$$(2 \times \underbrace{2)\,(2}_{} \times 3) = 2 \times 3$$

- Inside numbers must be equal (otherwise AB is undefined)
- Outside numbers give the size of AB

Ex: A is 2×3

B is 3×1

Size of AB and BA?

AB: $(2 \times \underbrace{3)\,(3}_{\checkmark} \times 1) = 2 \times 1$

$$\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$$

BA is undefined

$$(3 \times 1)\ (2 \times 3)$$

$\neq$

$$\begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix} \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

$\boxed{\text{FACT: } AB \neq BA \text{ in general}}$

Ex:
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 7 \\ 11 & 13 \\ 17 & 19 \end{bmatrix}$$

Why do we multiply like this?

Consider
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1x + 2y \\ 3x + 4y \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$\begin{cases} x + 2y = 5 \\ 3x + 4y = 6 \end{cases}$$

So we can study/solve systems.

System of equations:

$$A \vec{x} = \vec{b}$$

Coefficients

Variables (column)

Constants (column)