Week 4 Monday

January 27, 2020 7:55 AM

Test Friday

1.1–1.3, Cross Product, 2.1–2.2 (6 Questions)

No Formula Sheet

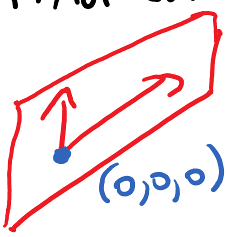
Practice Problems www.leahhoward.com

No office Hours Wed.

Review Thursday

2.3 Span and Linear Independence

<u>Ex</u>: Find an equation for span $\left(\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\3\\6\end{bmatrix}\right)$



<u>Vector form</u>

$\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} + s\begin{bmatrix}1\\0\\1\end{bmatrix} + t\begin{bmatrix}1\\3\\6\end{bmatrix}$$

ALTERNATIVELY:

<u>General form</u>

$$\vec{n} = \begin{bmatrix}1\\0\\1\end{bmatrix} \times \begin{bmatrix}1\\3\\6\end{bmatrix} = \begin{bmatrix}-3\\-5\\3\end{bmatrix}$$

1 0 1 1 0

1 3 6 1 3

$$-3x - 5y + 3z = d$$

Sub $(0,0,0)$: $\quad 0 = d$

$$\boxed{-3x - 5y + 3z = 0}$$

<u>Ex</u>: a) Show that $\text{span}\left(\begin{bmatrix}1\\3\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix}\right) = \mathbb{R}^2$

Let $\begin{bmatrix}a\\b\end{bmatrix}$ be any vector in $\mathbb{R}^2$

Show that $c_1\begin{bmatrix}1\\3\end{bmatrix} + c_2\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}$

has a solution c_1, c_2

$$\begin{array}{c} \begin{array}{cc} c_1 & c_2 \end{array} \\ \left[\begin{array}{cc|c} 1 & 2 & a \\ 3 & 1 & b \end{array}\right] \end{array}$$

$$R_2 - 3R_1 \quad \left[\begin{array}{cc|c} 1 & 2 & a \\ 0 & -5 & b-3a \end{array}\right] \text{ REF}$$

System is solvable

b) Write $\begin{bmatrix}a\\b\end{bmatrix}$ as a linear combination of $\begin{bmatrix}1\\3\end{bmatrix}$ and $\begin{bmatrix}2\\1\end{bmatrix}$.

Let $c_1\begin{bmatrix}1\\3\end{bmatrix} + c_2\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}$

$\vdots$

$$\begin{array}{c} \begin{array}{cc} c_1 & c_2 \end{array} \\ \left[\begin{array}{cc|c} 1 & 2 & a \\ 0 & -5 & b-3a \end{array}\right] \end{array} \to \text{RREF}$$

$$\left[\begin{array}{cc|c} 0 & -5 & b-3a \end{array}\right]$$

$$\frac{R_2}{-5} \quad \left[\begin{array}{cc|c} 1 & 2 & a \\ 0 & 1 & -\frac{1}{5}(b-3a) \end{array}\right]$$

$$R_1 - 2R_2 \quad \begin{array}{cc} C_1 & C_2 \end{array} \left[\begin{array}{cc|c} 1 & 0 & \frac{2b-a}{5} \\ 0 & 1 & \frac{3a-b}{5} \end{array}\right]$$

$$\boxed{\begin{aligned} & a + \tfrac{2}{5}(b-3a) \\ &= -\tfrac{1}{5}a + \tfrac{2}{5}b \end{aligned}}$$

RREF ✓

$$c_1 = \frac{2b-a}{5} \qquad c_2 = \frac{3a-b}{5}$$

$$\# \begin{bmatrix} 1 \\ 3 \end{bmatrix} + \# \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\frac{2b-a}{5} \begin{bmatrix} 1 \\ 3 \end{bmatrix} + \frac{3a-b}{5} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \quad ✓$$

Is the system solvable? Go to REF

Solve the system? Go to RREF

Def

Given $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$, Consider

$$c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_n\vec{v}_n = \vec{0}$$

Only solution is $c_1 = c_2 = \dots = c_n = 0$

$c_1 = c_2 = \dots = c_n = 0$ and other solutions

Vectors are <u>linearly independent</u>

Vectors are <u>linearly dependent</u>

<u>Ex</u>: a) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \end{bmatrix}$ are linearly <u>dependent</u>

$$-3\begin{bmatrix} 0 \\ 1 \end{bmatrix} + -1\begin{bmatrix} 2 \\ 4 \end{bmatrix} + 1\begin{bmatrix} 2 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

b) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ are linearly <u>dependent</u>

$$0\begin{bmatrix} 1 \\ 0 \end{bmatrix} + -2\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 1\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

c) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 7 \\ 7 \end{bmatrix}$ are lin. <u>dependent</u>

$$1\begin{bmatrix} 0 \\ 0 \end{bmatrix} + 0\begin{bmatrix} 7 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Alternative terminology:

$\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 7 \end{bmatrix} \right\}$ is lin. dep.

<u>Ex</u>: Are $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$ linearly independent?

Let $c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{array}{cccc} c_1 & c_2 & c_3 & \\ \multicolumn{4}{c}{\left[\begin{array}{ccc|c} \Big(\Big) & \Big(\Big) & \Big(\Big) & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \end{array}\right]} \end{array}$$

$c_1 = c_2 = c_3 = 0$ (trivial sol.) → Yes

trivial sol. and other solutions → No