Week 3 Wednesday

January 22, 2020 9:04 AM

2.2 Solving Systems Cont'd

Gaussian Elimination

Coefficients → REF

Solve by back-substitution

Ex: Solve by Gaussian Elimination

$$\begin{array}{ccc|c} x & y & z & \# \\ \hline 1 & 2 & 1 & 6 \\ 2 & 2 & 0 & 8 \\ 0 & 3 & 1 & 8 \end{array}$$

$R_2 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 0 & -2 & -2 & -4 \\ 0 & 3 & 1 & 8 \end{array}\right]$$

$\frac{R_2}{-2}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & 3 & 1 & 8 \end{array}\right]$$

$R_3 - 3R_2$

$$\begin{array}{ccc|c} x & y & z & \\ \hline 1 & 2 & 1 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -2 & 2 \end{array}$$

REF ✓

Back-substitution:

$-2z = 2 \rightarrow z = -1$

$y + z = 2 \rightarrow y - 1 = 2 \rightarrow y = 3$

$x + 2y + z = 6 \rightarrow x + 6 - 1 = 6 \rightarrow x = 1$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$

Def

A matrix is in reduced row-echelon form (RREF) if:

1) matrix is in REF
2) the leading nonzero entry of each row is 1
3) leading 1's have 0's everywhere else in their column

Ex:

$\left[\begin{array}{ccc|c} 1 & 0 & -3 & \\ 0 & 1 & 3 & \\ 0 & 0 & 0 & \end{array}\right]$ RREF (and REF)

$\left[\begin{array}{ccc|c} 1 & 0 & -3 & \\ 0 & 2 & 3 & \\ 0 & 0 & 0 & \end{array}\right]$ REF not RREF

$\left[\begin{array}{ccc|c} 1 & 1 & -3 & \\ 0 & 1 & 3 & \\ 0 & 0 & 0 & \end{array}\right]$ REF not RREF

DEF

Gauss-Jordan Elimination:

Coefficients → RREF

Ex: Solve by G-J Elimination

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 3 & 3 & 3 & 15 \\ 5 & 7 & 9 & 29 \end{array}\right]$$

$R_2 - 3R_1$, $R_3 - 5R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 0 & -3 & -6 & -6 \\ 0 & -3 & -6 & -6 \end{array}\right]$$

Current Row ± # (Pivot Row)

$\frac{R_2}{-3}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & -6 & -6 \end{array}\right]$$

x y z

$R_1 - 2R_2$, $R_3 + 3R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

RREF ✓

Circle leading 1 in each row

Columns without circles are free variables

$\boxed{z = k}$ ← "parameter"

$x - z = 3 \rightarrow x = z + 3 \rightarrow \boxed{x = k + 3}$

$y + 2z = 2 \rightarrow y = -2z + 2 \rightarrow \boxed{y = -2k + 2}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} k + \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$$

k: any real #

Ex: Solve by Gauss-Jordan elimination

$$\begin{array}{c} x \quad y \quad z \\ \left[\begin{array}{ccc|c} \textcircled{1} & 1 & -6 & 17 \\ 2 & 2 & -8 & 22 \\ 3 & 3 & -14 & 39 \end{array}\right] \end{array}$$

$R_2 - 2R_1$, $R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -6 & 17 \\ 0 & 0 & \textcircled{4} & -12 \\ 0 & 0 & 4 & -12 \end{array}\right]$$

$\frac{R_2}{4}$

$$\left[\begin{array}{ccc|c} 1 & 1 & -6 & 17 \\ 0 & 0 & \textcircled{1} & -3 \\ 0 & 0 & 4 & -12 \end{array}\right]$$

$R_1 + 6R_2$

$R_3 - 4R_2$

$$\begin{array}{c} x \quad y \quad z \\ \left[\begin{array}{ccc|c} \textcircled{1} & 1 & 0 & -1 \\ 0 & 0 & \textcircled{1} & -3 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

RREF ✓

$$\boxed{y = k}$$

$$x + y = -1 \rightarrow \boxed{x = -k - 1}$$

$$\boxed{z = -3}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} k + \begin{bmatrix} -1 \\ 0 \\ -3 \end{bmatrix}$$