Week 3 Tuesday

January 21, 2020 8:45 AM

2.1 Linear Systems Cont'd

$$\begin{cases} 2x - 3y = 8 \\ -4x + 6y = 20 \end{cases}$$

Coefficient matrix $\begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}$

Augmented matrix $\left[\begin{array}{cc|c} 2 & -3 & 8 \\ -4 & 6 & 20 \end{array}\right]$

3 Elementary Row Operations

1) Swap 2 Rows
2) Multiply / Divide a Row
3) Current Row $\pm$ # (Pivot Row)

Goal: Get $\left[\begin{array}{cc|c} 1 & 0 & \\ 0 & 1 & \end{array}\right]$

or as close as possible

<u>Ex</u>: Solve the system above

$$\left[\begin{array}{cc|c} 2 & -3 & 8 \\ -4 & 6 & 20 \end{array}\right]$$

Get a 1, "the pivot"

$$\frac{R_1}{2} \quad \left[\begin{array}{cc|c} 1 & -\frac{3}{2} & 4 \\ -4 & 6 & 20 \end{array}\right]$$

Get 0's in rest of Column 1

$$R_2 + 4R_1 \quad \begin{array}{ccc} x & y & \# \\ \left[\begin{array}{cc|c} 0 & 0 & 36 \end{array}\right] \end{array}$$

$$0x + 0y = 36 \quad \text{(impossible)}$$

System has no solution

FACT

If you see $[\,0 \;\; 0 \,|\, \text{nonzero}\,]$ or

$[\,0 \;\; 0 \;\; 0 \,|\, \text{nonzero}\,]$

then system has no solution.

<u>Ex:</u> Solve $\begin{array}{ccc} x & y & \# \\ \left[\begin{array}{cc|c} 2 & -3 & 8 \\ -4 & 6 & -16 \end{array}\right] \end{array}$

Get a 1

$$\frac{R_1}{2} \quad \left[\begin{array}{cc|c} 1 & -\frac{3}{2} & 4 \\ -4 & 6 & -16 \end{array}\right]$$

Get 0's in rest of Column 1

$$R_2 + 4R_1 \quad \begin{array}{ccc} x & y & \# \\ \left[\begin{array}{cc|c} 1 & -\frac{3}{2} & 4 \\ 0 & 0 & 0 \end{array}\right] \end{array} \quad \text{No INFO}$$

Column for y has no "1"

y is a free variable

$$\boxed{y = t} \leftarrow \text{any real \#}$$

$$x - \frac{3}{2}y = 4 \rightarrow x = \frac{3}{2}y + 4 \rightarrow \boxed{x = \frac{3}{2}t + 4}$$

$$\begin{cases} x = \frac{3}{2}t + 4 \\ y = t \end{cases}$$

$$\boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix} t + \begin{bmatrix} 4 \\ 0 \end{bmatrix}}$$

System has infinitely-many solutions

Ex: Solve

$$\begin{array}{ccc} x & y & \# \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 2 & 3 & 4 \\ 3 & 4 & 7 \end{array}\right]$$

Get a 1 ✓

Get 0's in rest of Column 1

$$\begin{array}{c} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 3 & -6 \\ 0 & 4 & -8 \end{array}\right]$$

Get a 1

$$\frac{R_2}{3} \quad \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 4 & -8 \end{array}\right]$$

Get 0's in Column 2

$$R_3 - 4R_2 \quad \begin{array}{c} \begin{array}{ccc} x & y & \# \end{array} \\ \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array}\right] \end{array} \leftarrow \text{No INFO}$$

$x = 5$

$y = -2$

1 UNIQUE SOLUTION

$$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{array}\right]$$

System has no solution

$R_2 - \frac{3}{2}R_1 \quad \left[\begin{array}{c|c} 2 & \\ 3 & \end{array}\right]$ is allowed, but inconvenient

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Ex: Solve by <u>back-substitution</u>:

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \# \end{array} \\ \left[\begin{array}{ccc|c} 4 & 1 & 1 & 15 \\ 0 & 3 & 5 & 29 \\ 0 & 0 & 2 & 8 \end{array}\right] \end{array}$$

Start @ bottom

$2z=8 \rightarrow z=4$

$3y+5z=29 \rightarrow 3y+20=29 \rightarrow y=3$

$4x+y+z=15 \rightarrow 4x+3+4=15 \rightarrow x=2$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

2.2 Solving Systems

A matrix is in <u>row-echelon form</u> (REF) if:

1) any zero rows are at the bottom AND

2) the leading nonzero entries of each row move down and right

$$\left[\begin{array}{ccc|c} 6 & 0 & -1 & \\ 0 & 0 & 3 & \\ 0 & 0 & 0 & \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 3 & \\ 0 & 2 & -4 & \\ 0 & 0 & 0 & \end{array}\right]$$

REF REF

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 3 & \end{array}\right]$$

REF