Test Fri Jan 31^{st}
1.1 – 1.3, Cross Product, 2.1 – 2.2
(6 Questions)
No Formula Sheet
Practice Problems www.leahhoward.com

(1.4) Cross Product Cont'd

Ex: $\vec{u} = [1, -2, 3]$ $\quad \vec{v} = [-4, 5, 6]$
Find $\vec{u} \times \vec{v}$

Method I

$$\begin{array}{ccccc} 1 & -2 & 3 & 1 & -2 \\ -4 & 5 & 6 & -4 & 5 \end{array}$$

$$\vec{u} \times \vec{v} = [-27, -18, -3]$$

Method II

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 3 \\ -4 & 5 & 6 \end{vmatrix}$$

$$\begin{bmatrix} + & - & + \\ & & \end{bmatrix}$$

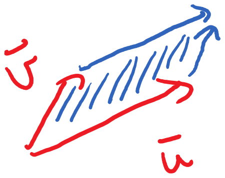
$$= \vec{i} \begin{vmatrix} -2 & 3 \\ 5 & 6 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & 3 \\ -4 & 6 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & -2 \\ -4 & 5 \end{vmatrix}$$

$$= \vec{i}(-27) - \vec{j}(18) + \vec{k}(-3)$$

$$= [-27, -18, -3]$$

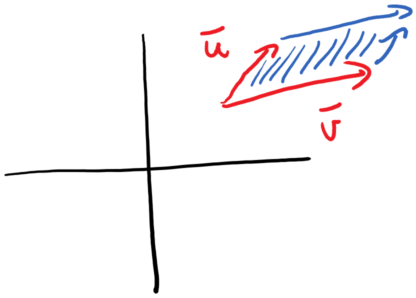
3 Geometry Formulas

1) Area (parallelogram in $\mathbb{R}^3$) $= \| \vec{u} \times \vec{v} \|$

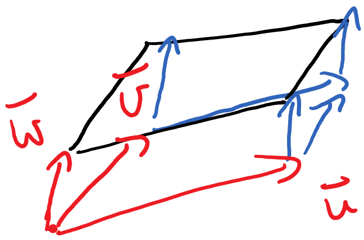


2) Area (parallelogram in $\mathbb{R}^2$)

$= \text{absolute value of } \det \begin{bmatrix} \vec{u} \\ \vec{v} \end{bmatrix}$



3)



slanted box = parallelepiped

$$\boxed{\begin{aligned} &V(\text{parallelepiped}) \\ &\quad = \text{absolute value of } \det \begin{bmatrix} \vec{u} \\ \vec{v} \\ \vec{w} \end{bmatrix} \end{aligned}}$$

<u>Ex</u>: Do vectors [1, 4, 7], [2, 5, 9] and [1, -2, -3] lie in a plane?

$\boxed{\text{Yes if and only if } V(\text{parallelepiped}) = 0}$

Yes if and only if V(parallelepiped) = 0

$$V(\text{slanted box}) = \left| \begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 9 \\ 1 & -2 & -3 \end{vmatrix} \right|$$

$$\begin{bmatrix} + & - & + \\ & & \end{bmatrix}$$

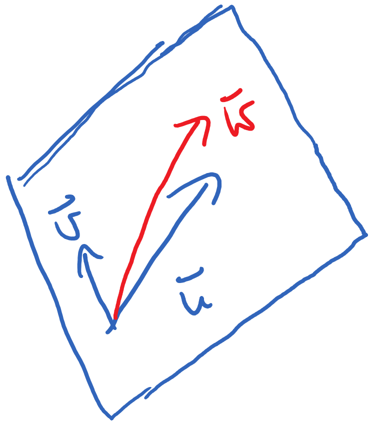
$$= \left| 1 \begin{vmatrix} 5 & 9 \\ -2 & -3 \end{vmatrix} - 4 \begin{vmatrix} 2 & 9 \\ 1 & -3 \end{vmatrix} + 7 \begin{vmatrix} 2 & 5 \\ 1 & -2 \end{vmatrix} \right|$$

$$= \left| 1(3) - 4(-15) + 7(-9) \right|$$

$$= |0|$$

$$= 0$$

$\boxed{\text{Yes}}$



<u>Ex</u>: Find the area of the parallelogram determined by [1,6] and [3,5].

$$\text{area} = \left| \begin{vmatrix} 1 & 6 \\ 3 & 5 \end{vmatrix} \right|$$

$$= |-13|$$

$$= 13$$

2.1 Linear Systems

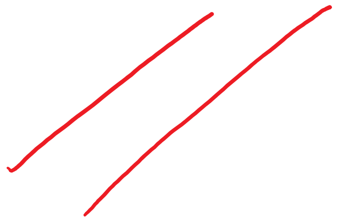
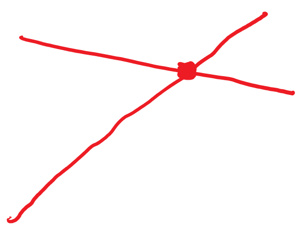
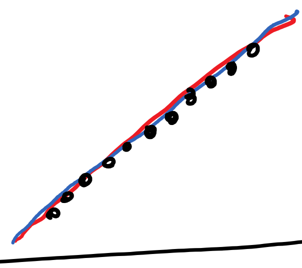
Linear equation: $ax + by + cz = d$

a, b, c, d: real numbers

Linear system
$$\begin{cases} 2x - y + 5z = 3 \\ x + 2y + 3z = 4 \end{cases}$$

FACT

A system can have:

NO SOLUTION	1 UNIQUE SOLUTION	INFINITELY-MANY SOLUTIONS
		

"inconsistent system"

"consistent systems" (solvable systems)

Ex: Solve by elimination

$$\begin{cases} 2x + 6y = -14 \\ -3x + 3y = -15 \end{cases}$$

$$\begin{array}{c} \begin{array}{ccc} x & y & \# \end{array} \\ \left[\begin{array}{cc|c} 2 & 6 & -14 \\ -3 & 3 & -15 \end{array}\right] \end{array}$$

Goal: $\left[\begin{array}{cc|c} 1 & 0 & \\ 0 & 1 & \end{array}\right]$

Get a 1

$$\frac{R_1}{2} \quad \left[\begin{array}{cc|c} 1 & 3 & -7 \\ -3 & 3 & -15 \end{array}\right]$$

Get 0

$$R_2 + 3R_1 \quad \left[\begin{array}{cc|c} 1 & 3 & -7 \\ \underline{\underline{0}} & 12 & -36 \end{array}\right]$$

Get a 1

$$R_2/12 \quad \left[\begin{array}{cc|c} 1 & 3 & -7 \\ 0 & 1 & -3 \end{array}\right]$$

Get a 0

$$\begin{array}{c} R_1 - 3R_2 \quad \left[\begin{array}{cc|c} 1 & \underline{\underline{0}} & 2 \\ 0 & 1 & -3 \end{array}\right] \\ \begin{array}{ccc} x & y & \# \end{array} \end{array}$$

0

$1x + 0y = 2 \rightarrow$ $x = 2$

$y = -3$