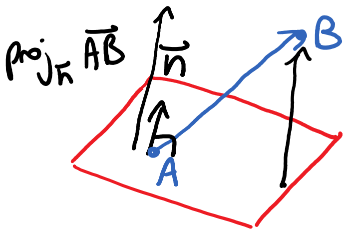
Week 2 Tuesday

January 14, 2020 9:10 AM

1.3 Lines and Planes Cont'd

<u>Ex</u>: Find the distance between $B=(1,3,3)$ and the plane $x+y+2z=7$.



Choose any point on plane

$A=(7,0,0)$

$$\text{Distance} = \|\text{proj}_{\vec{n}}\vec{AB}\|$$

$$\vec{AB} = \begin{bmatrix}-6\\3\\3\end{bmatrix}$$

$$\vec{n} = \begin{bmatrix}1\\1\\2\end{bmatrix}$$

$$\text{proj}_{\vec{n}}\vec{AB} = \frac{\vec{n}\cdot\vec{AB}}{\|\vec{n}\|^2}\vec{n}$$

$$= \frac{3}{6}\begin{bmatrix}1\\1\\2\end{bmatrix}$$

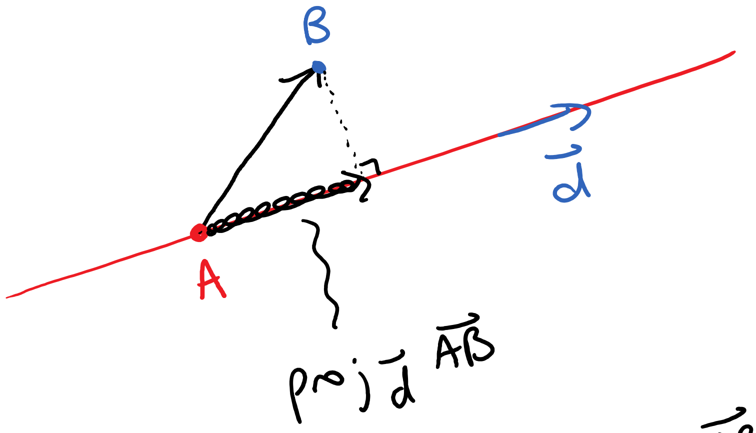
$$= \frac{1}{2}\begin{bmatrix}1\\1\\2\end{bmatrix}$$

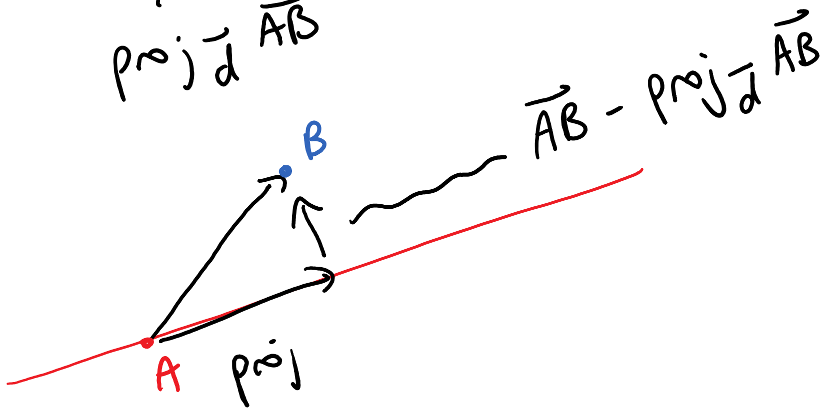
$$\text{Distance} = \left\|\frac{1}{2}\begin{bmatrix}1\\1\\2\end{bmatrix}\right\|$$

$$= \frac{\sqrt{6}}{2}$$

$$\boxed{\|c\vec{v}\| = |c|\,\|\vec{v}\|}$$

Ex: Find the distance between $B=(1,1,0)$ and the line through $A=(0,1,2)$ with $\vec{d} = \begin{bmatrix}1\\0\\1\end{bmatrix}$.





$$\text{distance} = \left\| \vec{AB} - \text{proj}_{\vec{d}}\,\vec{AB} \right\|$$

$$\vec{AB} = \begin{bmatrix}1\\0\\-2\end{bmatrix} \qquad \vec{d} = \begin{bmatrix}1\\0\\1\end{bmatrix}$$

$$\text{proj}_{\vec{d}}\,\vec{AB} = \frac{\vec{AB}\cdot\vec{d}}{\|\vec{d}\|^2}\,\vec{d}$$

$$= \frac{-1}{2}\begin{bmatrix}1\\0\\1\end{bmatrix}$$

$$\vec{AB} - \text{proj}_{\vec{d}}\,\vec{AB} = \begin{bmatrix}1\\0\\-2\end{bmatrix} + \frac{1}{2}\begin{bmatrix}1\\0\\1\end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{2} \\ 0 \\ -\frac{3}{2} \end{bmatrix}$$

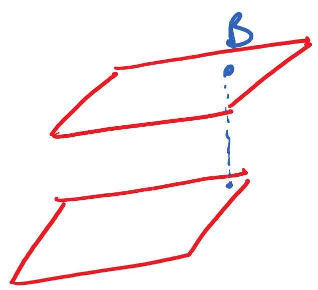
$$= \frac{1}{2} \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$$

$$\text{distance} = \left\| \frac{1}{2} \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix} \right\|$$

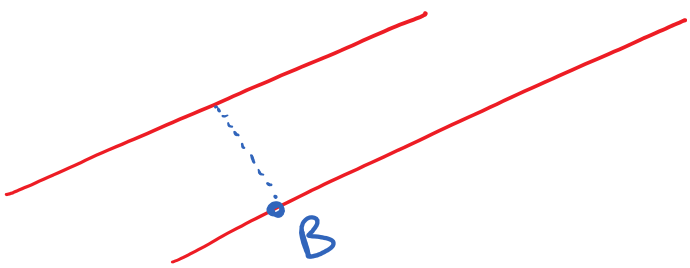
$$= \frac{1}{2}\sqrt{18} \quad \text{or} \quad \frac{3\sqrt{2}}{2}$$

<u>Comments</u>:

1) For distance between parallel planes, choose any point on either plane as B.



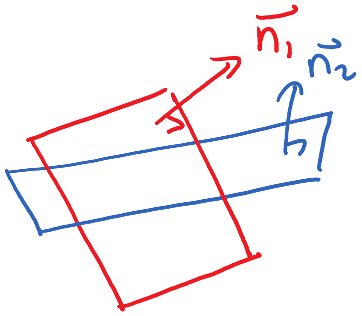
2) For distance between parallel lines, choose any point on either line as B.

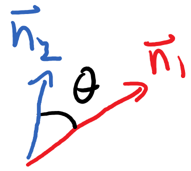


Def

The <u>angle between planes</u> is the angle between the normals.

normal vectors





Def

<u>Parallel planes</u> have parallel normals

<u>Perpendicular "</u> " perpendicular "

(1.4) <u>Cross Product</u>

Cross product $\bar{u} \times \bar{v}$ is defined for $\bar{u}, \bar{v}$ in $\mathbb{R}^3$

<u>Ex:</u> $\bar{u} = [1, 2, 1]$ $\bar{v} = [3, -1, 4]$

$$\begin{array}{ccccc} 1 & 2 & 1 & 1 & 2 \\ 3 & -1 & 4 & 3 & -1 \end{array}$$

$$\vec{u} \times \vec{v} = [\,2(4) - 1(-1),\ \ 1(3) - 1(4),\ \ 1(-1) - 2(3)\,]$$

$$= [\,9,\ -1,\ -7\,]$$

Ex: a) Compute $\vec{v} \times \vec{u}$

$$\begin{array}{ccccc} 3 & -1 & 4 & 3 & -1 \\ 1 & 2 & 1 & 1 & 2 \end{array}$$

$$\vec{v} \times \vec{u} = [-9,\ 1,\ 7]$$

b) Compute $(\vec{u} \times \vec{v}) \cdot \vec{u}$

$\vec{u} \times \vec{v} = [9, -1, -7]$ $\quad \vec{u} = [1, 2, 1]$

$$(\vec{u} \times \vec{v}) \cdot \vec{u} = 0$$

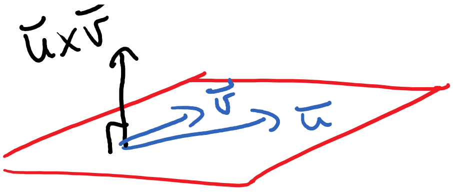
FACTS

1) $\vec{v} \times \vec{u} = -(\vec{u} \times \vec{v})$

2) $\vec{u} \times \vec{v}$ is perpendicular to both $\vec{u}$ and $\vec{v}$

FACT

$\vec{u} \times \vec{v}$ is a normal vector for the plane containing $\vec{u}$ and $\vec{v}$



Right Hand Rule determines the direction of $\bar{u}\times\bar{v}$