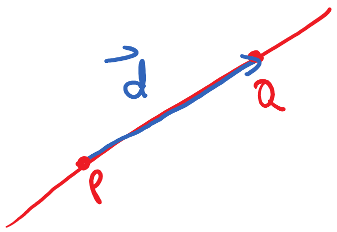
Week 2 Monday
January 13, 2020 7:56 AM

1.3 Lines and Planes Cont'd

Lines in $\mathbb{R}^3$

Ex: Vector and parametric form of the line through $P(2,1,12)$ and $Q(0,-3,6)$?



$$\vec{d} = \vec{PQ} = \begin{bmatrix} -2 \\ -4 \\ -6 \end{bmatrix}$$

Vector form $\quad \vec{x} = \vec{p} + t\vec{d}$

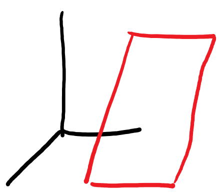
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 12 \end{bmatrix} + t \begin{bmatrix} -2 \\ -4 \\ -6 \end{bmatrix}$$

Parametric Form

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 12 \end{bmatrix} + \begin{bmatrix} -2t \\ -4t \\ -6t \end{bmatrix}$$

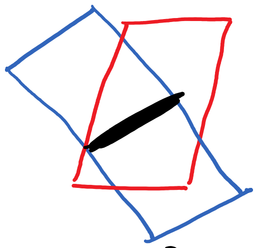
$$\begin{cases} x = 2 - 2t \\ y = 1 - 4t \\ z = 12 - 6t \end{cases}$$

FACT
$ax + by + cz = d$ is a plane in $\mathbb{R}^3$



Plane = infinite, flat surface

Note: A line in $\mathbb{R}^3$ is an intersection of 2 planes



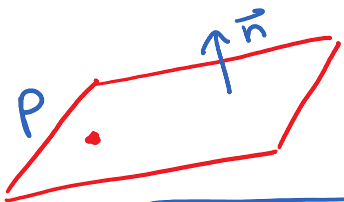
General form of a line in $\mathbb{R}^3$:

$$\begin{cases} 2x - y + z = 3 \\ x + y - 4z = 9 \end{cases}$$ (for example)

General and normal form are not useful for lines in $\mathbb{R}^3$

Planes in $\mathbb{R}^3$

Ex: Normal and general forms for the plane through $P(1,-1,3)$ with normal $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$?



Normal form $\quad \vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$

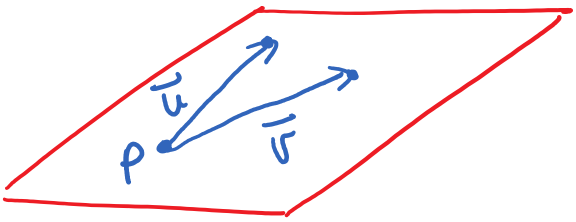
$$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

General form $\quad 1x + 1y + 2z = 6$

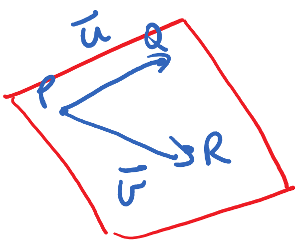
Def
Vector form of a plane in $\mathbb{R}^3$ is

$$\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$$

where $\vec{u}, \vec{v}$ are <u>nonparallel</u> direction vectors
s, t are any real numbers



<u>Ex</u>: Vector and parametric form of the plane through $P(6,0,0)$, $Q(0,6,0)$ and $R(0,0,3)$?



Direction vectors $\vec{u} = \overrightarrow{PQ} = \begin{bmatrix} -6 \\ 6 \\ 0 \end{bmatrix}$

$\vec{v} = \overrightarrow{PR} = \begin{bmatrix} -6 \\ 0 \\ 3 \end{bmatrix}$

Not parallel ✓
(not multiples of one another)

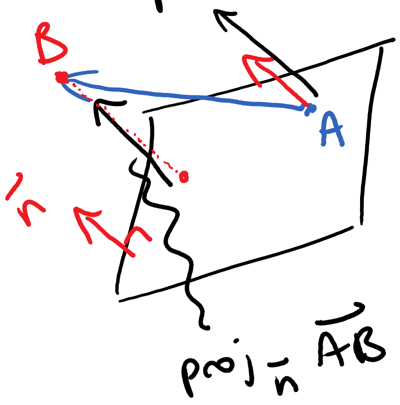
Vector Form $\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -6 \\ 6 \\ 0 \end{bmatrix} + t\begin{bmatrix} -6 \\ 0 \\ 3 \end{bmatrix}$$

Parametric Form
$$\begin{cases} x = 6 - 6s - 6t \\ y = 6s \\ z = 3t \end{cases}$$

	Lines in $\mathbb{R}^2$	Lines in $\mathbb{R}^3$	Planes in $\mathbb{R}^3$
GENERAL	$ax+by=c$	No	$ax+by+cz=d$
NORMAL	$\vec{n}\cdot\vec{x} = \vec{n}\cdot\vec{p}$	NO	$\vec{n}\cdot\vec{x} = \vec{n}\cdot\vec{p}$
VECTOR	$\vec{x} = \vec{p} + t\vec{d}$	$\vec{x} = \vec{p} + t\vec{d}$	$\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$
PARAMETRIC	$\begin{cases} x = \\ y = \end{cases}$	$\begin{cases} x = \\ y = \\ z = \end{cases}$	$\begin{cases} x = \\ y = \\ z = \end{cases}$

Ex: Find the distance between point B and plane?



$$\text{distance} = \| \text{proj}_{\bar{n}} \overrightarrow{AB} \|$$