Week 10 Wednesday

March 11, 2020 9:06 AM

4.3 Finding Eigenvalues Cont'd

Ex: A is 5×5

$$|A-\lambda I| = (7-\lambda)^3(9-\lambda)^2$$

Basis for $E_7 = \{[\ ]\}$

Basis for $E_9 = \{[\ ],[\ ]\}$

Eigenvalue	Algebraic Multiplicity	Geometric Mult.
$\lambda=7$	3	1
$\lambda=9$	2	2

FACT
For any eigenvalue,
geometric multiplicity $\le$ algebraic multiplicity

When geo. mult. = alg. mult. for all eigenvalues, matrix A has a useful property (Section 4.4)

5 Facts about Eigenvalues

① A is invertible if and only if 0 is not an eigenvalue of A

A is invertible

$\iff \det A \neq 0$

$\iff \det(A-0I)\neq 0$

$\iff$ 0 is not an eigenvalue of A

② If A is invertible and $A\vec{x} = \lambda\vec{x}$
then $\vec{x}$ is an eigenvector for A^{-1} with eigenvalue $\frac{1}{\lambda}$

$$A\vec{x} = \lambda\vec{x}$$

Mult by A^{-1}:

$$A^{-1}A\vec{x} = A^{-1}\lambda\vec{x}$$

$$\vec{x} = A^{-1}\lambda\vec{x}$$

$$\vec{x} = \lambda A^{-1}\vec{x} \qquad (\lambda \text{ is a constant})$$

$$A^{-1}\vec{x} = \frac{1}{\lambda}\vec{x}$$

eigenvector eigenvalue

③ If $A\vec{x} = \lambda\vec{x}$ then $A^n\vec{x} = \lambda^n\vec{x}$
for $n = 2, 3, 4, \ldots$

$$A\vec{x} = \lambda\vec{x}$$

Mult. by A:

$$A^2\vec{x} = A\lambda\vec{x} = \lambda A\vec{x} = \lambda\lambda\vec{x} = \lambda^2\vec{x}$$

Quick Ex: A is invertible and has $\lambda = -1, 3$

Eigenvalues of A^{-1}: $-1, \frac{1}{3}$

Eigenvalues of A^n: $(-1)^n, 3^n$

$\boxed{\text{④ If } A\vec{x} = \lambda\vec{x} \text{ then } \vec{x} \text{ is an eigenvector of } A+kI \text{ with eigenvalue } \lambda+k}$

$$(A+kI)\vec{x} = A\vec{x} + kI\vec{x}$$
$$= \lambda\vec{x} + k\vec{x}$$
$$= (\lambda+k)\vec{x}$$

⑤ Suppose A has eigenvectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m$ corresponding to $\lambda_1, \lambda_2, \dots, \lambda_m$.

$$A^k(c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_m\vec{v}_m) = c_1\lambda_1^k\vec{v}_1 + c_2\lambda_2^k\vec{v}_2 + \dots + c_m\lambda_m^k\vec{v}_m$$

- Generalization of Fact 3
- Coefficients are preserved

Ex: A is a 2×2 matrix with eigenvector $\vec{v}_1 = \begin{bmatrix}1\\2\end{bmatrix}$ corresponding to $\lambda_1 = -2$

" $\vec{v}_2 = \begin{bmatrix}2\\-1\end{bmatrix}$ " $\lambda_2 = 3$

Find $A^3\begin{bmatrix}11\\2\end{bmatrix}$