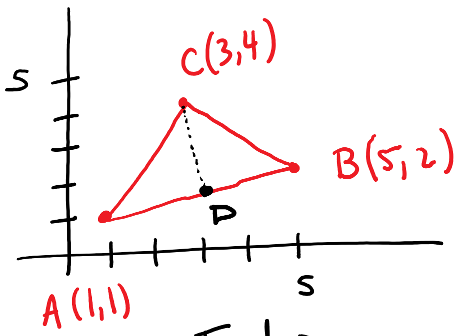
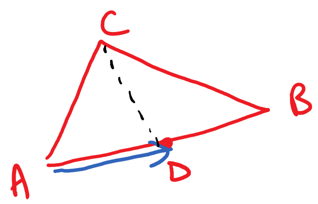
Week 1 Friday

January 10, 2020 9:39 AM

1.2 Cont'd



Find D



$$\vec{AD} = \text{proj}_{\vec{AB}} \vec{AC}$$

$$= \frac{\vec{AB}\cdot\vec{AC}}{\|\vec{AB}\|^2}\vec{AB}$$

$$\boxed{\begin{aligned}\vec{AB} &= [4,1] \\ \vec{AC} &= [2,3]\end{aligned}}$$

$$= \frac{11}{17}[4,1]$$

$$= \frac{1}{17}[44,11]$$

$$\vec{A} + \vec{AD} = \vec{D}$$

$$\vec{D} = [1,1] + \frac{1}{17}[44,11]$$

$$= \frac{1}{17}[17,17] + \frac{1}{17}[44,11]$$

$$\boxed{\begin{gathered}A = (1,1) \\ \vec{A} = [1,1] \\ \text{Formally, } \vec{A} = \vec{OA}\end{gathered}}$$

$= \frac{1}{17}[61, 28]$

$D = \left(\frac{61}{17}, \frac{28}{17}\right)$

$\approx (3.6, 1.6)$

1.3 Lines and Planes

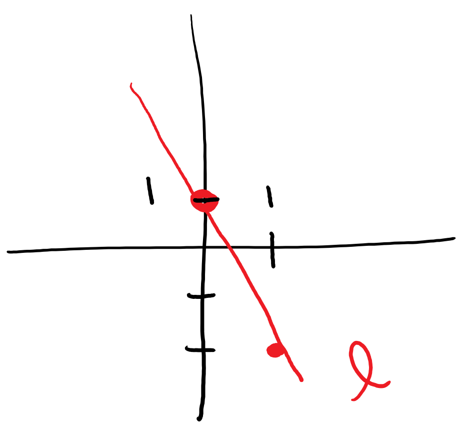
Lines in $\mathbb{R}^2$

Def
General form of a line in $\mathbb{R}^2$ is $ax+by=c$

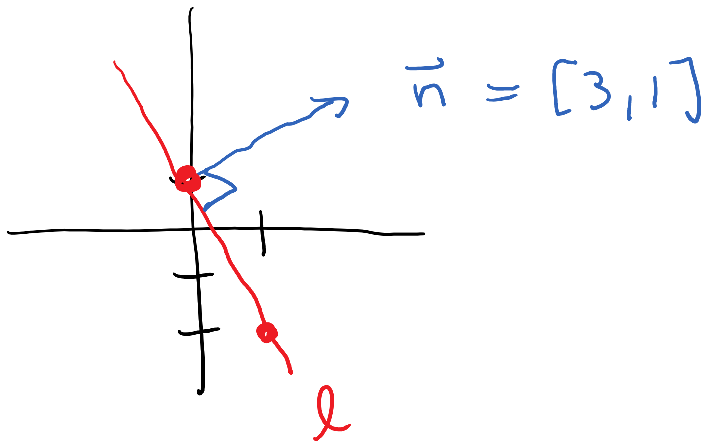
Ex: Line ℓ : $3x+y=1$

Set $x=0$: $y=1$ $\quad P(0,1)$

$x=1$: $y=-2$ $\quad Q(1,-2)$



Def
The <u>normal vector</u> $\vec{n}$ is orthogonal to ℓ
Its components are the coefficients of the general form.



Def
The <u>normal form</u> of a line in $\mathbb{R}^2$ is

$$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p} \quad \text{where}$$

$$\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

and $\vec{p}$ is the vectorization of any point on the line

<u>Ex</u>: For ℓ above,

$\vec{n} = [3,1]$

$P = (0,1) \qquad \vec{p} = [0,1]$

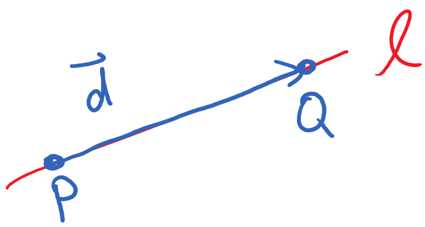
Normal Form $\quad \vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$

$$\begin{bmatrix}3\\1\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix}\cdot\begin{bmatrix}0\\1\end{bmatrix}$$

General Form $\qquad 3x+y=1$

Def

A <u>direction vector</u> for a line is $\vec{d}=\overrightarrow{PQ}$, where P and Q are points on the line.



Def

The <u>vector form</u> of a line in $\mathbb{R}^2$ is

$$\vec{x}=\vec{p}+t\vec{d} \qquad \text{where}$$

$$\vec{x}=\begin{bmatrix}x\\y\end{bmatrix}$$

$\vec{p}$ = vectorization of a point

t = any real #

<u>Ex</u>: Same ℓ: $3x+y=1$

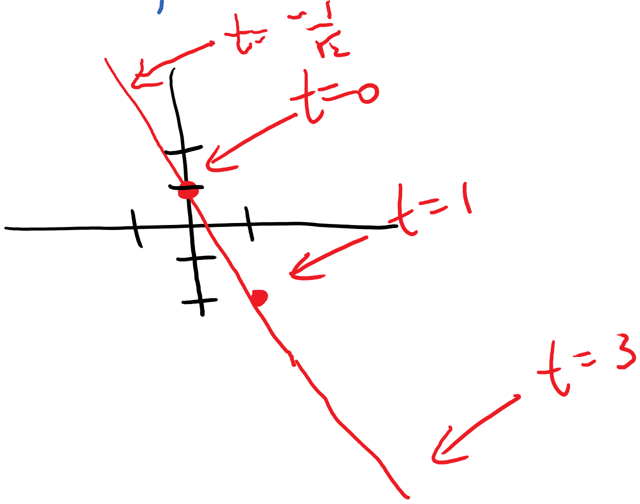
$P = (0,1)$ $\quad Q = (1,-2)$

$\vec{d} = \vec{PQ} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

Vector form $\quad \vec{x} = \vec{p} + t\vec{d}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

As t varies, the line is drawn



Def

Parametric form of a line in $\mathbb{R}^2$

$$\begin{cases} x = a + bt \\ y = c + dt \end{cases}$$

Ex:

Vector Form $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} t \\ -3t \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0+t \\ 1-3t \end{bmatrix}$$

Parametric Form $\begin{cases} x = 0+t \\ y = 1-3t \end{cases}$

Note: Forms are not unique

$3x+y=1$ and $6x+2y=2$
are the same line.

Summary: Lines in $\mathbb{R}^2$

General Form	$3x+y=1$
Normal	$\begin{bmatrix} 3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Vector Form

Parametric

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$\begin{cases} x = 0 + t \\ y = 1 - 3t \end{cases}$$