Coursepack: D2L or
www.leahhoward.com/251CP.pdf

Can print at Satellite Printshop

Do Sugg HW Section 1.1

Note: Coursepack uses bold font for vectors
$c$: scalar
$\mathbf{c}$: vector

---

## 1.2 Length and Angle

$\vec{u} = [1, 4, 2, -9]$ $\vec{v} = [2, 3, -2, -1]$

dot product $\vec{u} \cdot \vec{v} = 1(2) + 4(3) + 2(-2) + (-9)(-1)$

$= 19$

Ex: a) $[1, 5] \cdot [2, -3]$

$= 1(2) + 5(-3)$

$= -13$

b) $[1, 5] \cdot [2, -3, 0]$
is undefined

c) $[u_1, u_2] \cdot [u_1, u_2]$

$= u_1^2 + u_2^2$

**FACT**

3 Properties of the Dot Product

1) $\vec{u} \cdot \vec{u} \geq 0$

2) $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$ for all $\vec{u}, \vec{v}$

3) $\vec{u} \cdot \vec{u} = 0$ if and only if $\vec{u} = \vec{0}$

Means:

3a) if $\vec{u} \cdot \vec{u} = 0$ then $\vec{u} = \vec{0}$ LESS OBVIOUS

3b) if $\vec{u} = \vec{0}$ then $\vec{u} \cdot \vec{u} = 0$ OBVIOUS

Ex: Simplify

a) $(\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v})$

$= \vec{u} \cdot \vec{u} + \vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{u} + \vec{v} \cdot \vec{v}$

$= \vec{u} \cdot \vec{u} + 2\,\vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{v}$

b) $3\vec{u} \cdot (-2\vec{v} + 5\vec{w})$

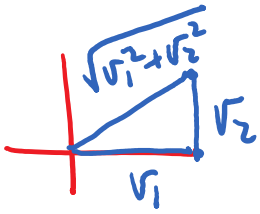
$= -6\,\vec{u} \cdot \vec{v} + 15\,\vec{u} \cdot \vec{w}$

**Def**

The length of $\vec{v} = [v_1, v_2, \dots, v_n]$

is $\| \vec{v} \| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$

Generalization of




Ex: a) $\| [1, 1, 1, -2] \|$

$= \sqrt{1+1+1+4}$

$= \sqrt{7}$

b) $\| [3, -1] \|$

$= \sqrt{9+1}$

$= \sqrt{10}$

Notice $[3,-1]\cdot[3,-1] = 9+1 = 10$

$\|[3,-1]\| = \sqrt{10}$

**Fact** $\vec{u}\cdot\vec{u} = \|\vec{u}\|^2$ for any vector $\vec{u}$

Ex: Let $\vec{v} = [v_1, v_2, v_3]$

Simplify $\|-3\vec{v}\|$

$$\|-3\vec{v}\| = \|[-3v_1, -3v_2, -3v_3]\|$$

$$= \sqrt{9v_1^2 + 9v_2^2 + 9v_3^2}$$

$$= \sqrt{9(v_1^2 + v_2^2 + v_3^2)}$$

$$= 3\sqrt{v_1^2 + v_2^2 + v_3^2}$$

$$= 3\|\vec{v}\|$$

**Fact** $\|c\vec{v}\| = |c|\,\|\vec{v}\|$

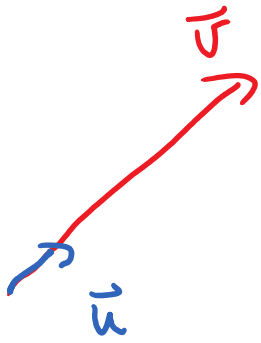
**Def**

A unit vector has length 1

To normalize a vector means find a unit vector in the same direction

**FACT**

$\vec{u} = \frac{1}{\|\vec{v}\|}\vec{v}$ has length 1 and same direction as $\vec{v}$



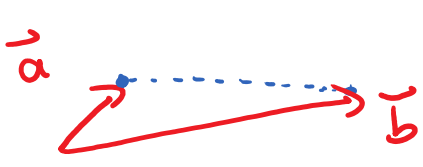

Ex: Normalize $\vec{v} = [4, -2, 1]$

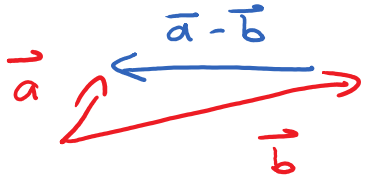
$$\frac{1}{\sqrt{21}}[4, -2, 1]$$

**FACT**

The distance between $\vec{a}$ and $\vec{b}$ is

$$d(\vec{a}, \vec{b}) = \|\vec{a} - \vec{b}\| \text{ or } \|\vec{b} - \vec{a}\|$$



**Ex:** Find distance between $[2,-1]$ and $[3,-6]$

$$d = \| [-1, 5] \|$$

$$= \sqrt{26}$$

**FACT** Triangle Inequality

$$\|\vec{u}+\vec{v}\| \leq \|\vec{u}\| + \|\vec{v}\| \quad \text{for any } \vec{u},\vec{v} \text{ in } \mathbb{R}^n$$