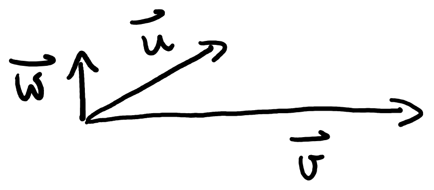
Week 1 Tuesday

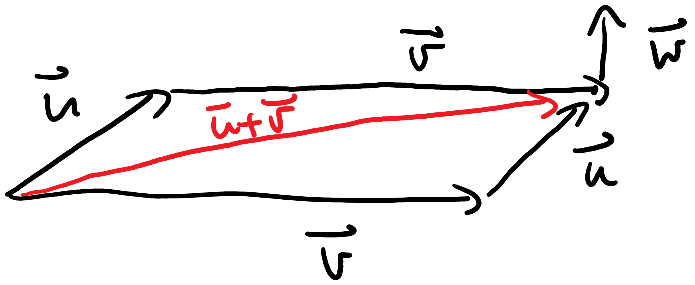
January 7, 2020 8:31 AM

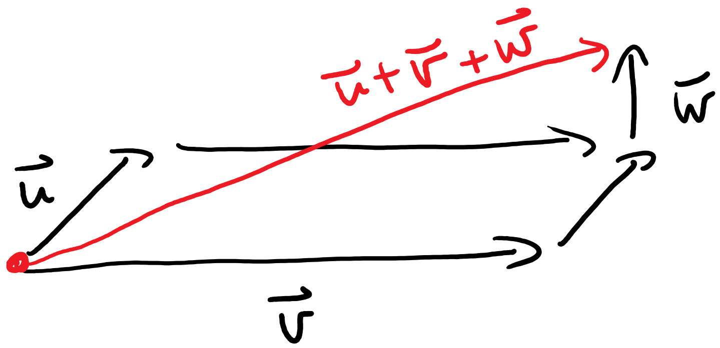
1.1 Geometry and Algebra of Vectors Cont'd

Ex:



Draw $\vec{u}+\vec{v}+\vec{w}$

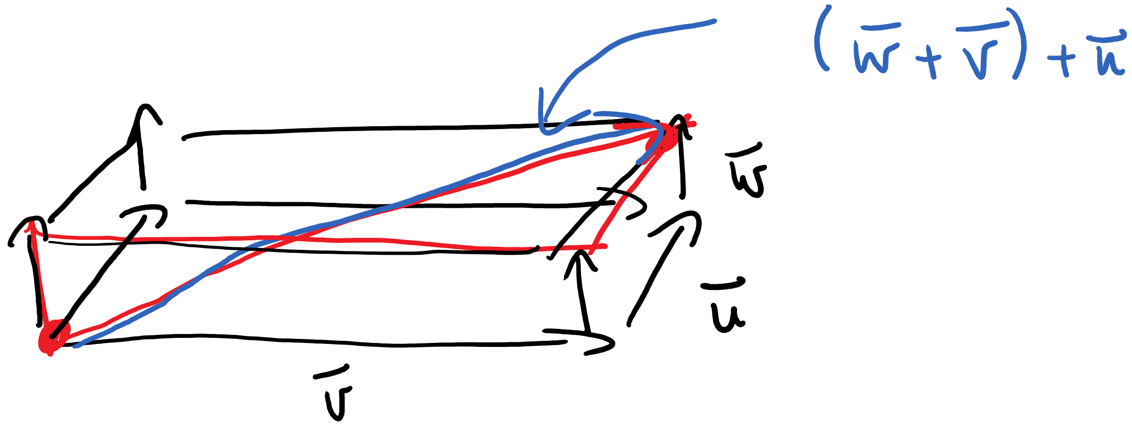




Order doesn't matter when adding vectors

$$(\vec{u}+\vec{v})+\vec{w}$$

$$=(\vec{w}+\vec{v})+\vec{u}$$

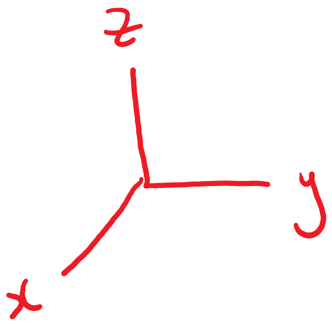


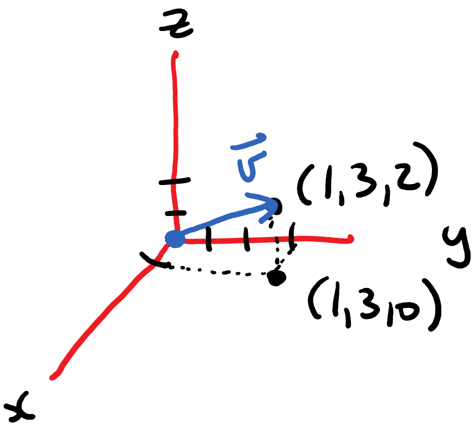
Notation: $\vec{v}$ in $\mathbb{R}^n$ means that $\vec{v}$ has n components, and each component is a real number.

Ex: $\vec{v} = [1, 3, 2]$

$\vec{v}$ is in $\mathbb{R}^3$

Draw $\vec{v}$





The zero vector $\vec{0}$

$\vec{0} = [0,0]$ in $\mathbb{R}^2$

$\vec{0} = [0,0,0]$ in $\mathbb{R}^3$

etc.

Useful for algebra

Ex: Let $\vec{u}$ be in $\mathbb{R}^2$

Show that $\vec{u} + (-\vec{u}) = \vec{0}$

Let $\vec{u} = [u_1, u_2]$

Start with more complicated side

$$\vec{u} + (-\vec{u}) = [u_1, u_2] + [-u_1, -u_2]$$

$$= [0, 0]$$

$$= \vec{0}$$

Ex: Solve for $\vec{x}$

$$7\vec{x} - \vec{a} = 3(\vec{a} + 4\vec{x})$$

Usual arithmetic rules apply

$$7\vec{x} - \vec{a} = 3\vec{a} + 12\vec{x}$$

$$-5\vec{x} = 4\vec{a}$$

$$\vec{x} = -\frac{4}{5}\vec{a}$$

<u>Ex</u>: $\vec{w} = -3\begin{bmatrix}1\\1\end{bmatrix} + 2\begin{bmatrix}0\\2\end{bmatrix}$

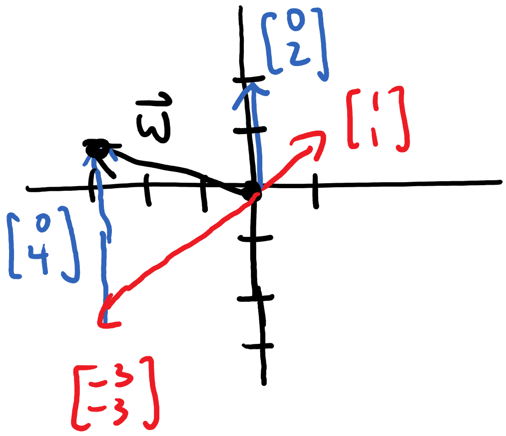
Terminology: " $\vec{w}$ is a <u>linear combination</u> of $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}0\\2\end{bmatrix}$, with coefficients -3 and 2"

a) Find $\vec{w}$ algebraically

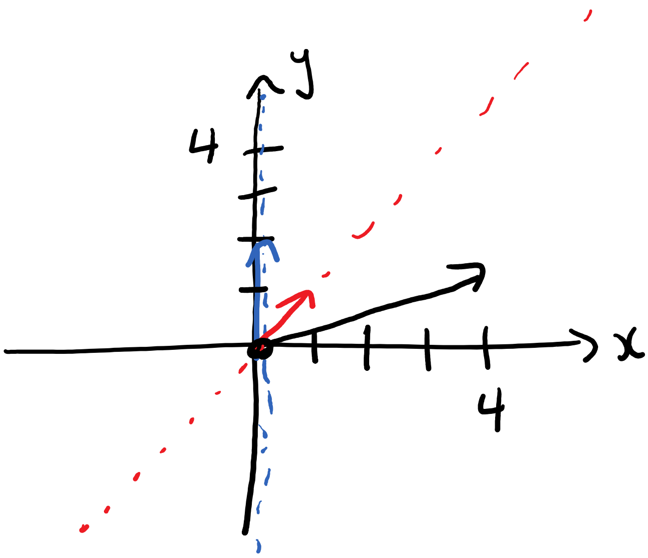
$$\vec{w} = \begin{bmatrix}-3\\-3\end{bmatrix} + \begin{bmatrix}0\\4\end{bmatrix}$$

$$= \begin{bmatrix}-3\\1\end{bmatrix}$$

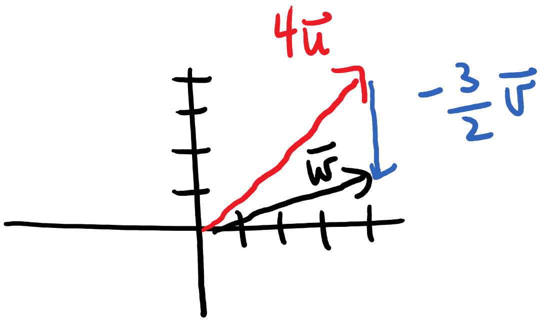
b) Find $\vec{w}$ geometrically



Ex: Write $\vec{w} = \begin{bmatrix}4\\1\end{bmatrix}$ as a linear combination of $\vec{u} = \begin{bmatrix}1\\1\end{bmatrix}$ and $\vec{v} = \begin{bmatrix}0\\2\end{bmatrix}$ by graphing.



Think of $\vec{u}$ and $\vec{v}$ as the axes.



$$\boxed{\begin{aligned} k(2) &= -3 \\ k &= \frac{-3}{2} \end{aligned}}$$

$$\bar{w} = 4\bar{u} - \frac{3}{2}\bar{v}$$

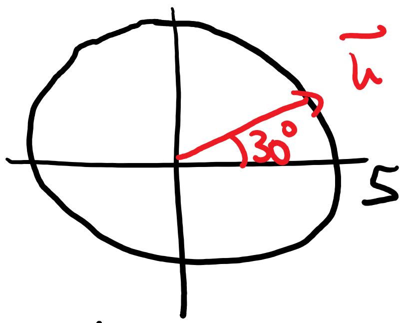
We'll do this algebraically in Ch. 2

$$\begin{bmatrix} 4 \\ 1 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

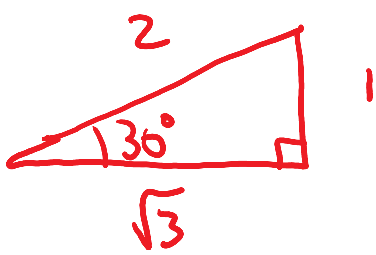
$\vdots$

<u>Ex</u>: TRIG

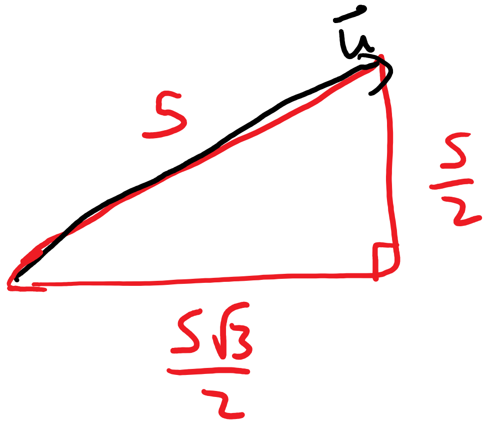
a)



Find $\vec{u}$

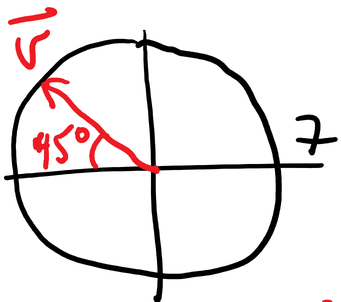


Multiply by $\frac{5}{2}$:

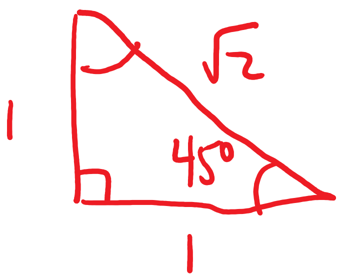


$$\bar{u} = \left[\frac{5\sqrt{3}}{2}, \frac{5}{2}\right]$$

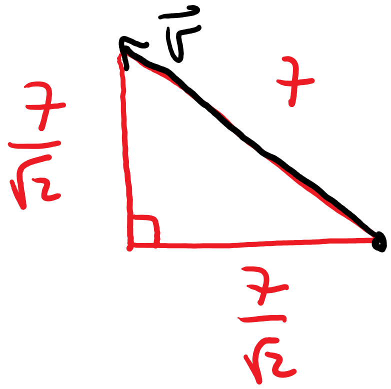
b)



Find $\bar{v}$



Multiply by $\frac{7}{\sqrt{2}}$:



Watch signs

$$\vec{v} = \left[-\frac{7}{\sqrt{2}}, \frac{7}{\sqrt{2}} \right]$$

$$\text{or } \left[\frac{-7\sqrt{2}}{2}, \frac{7\sqrt{2}}{2} \right]$$