Week 9 Thursday

March 5, 2020 9:41 AM

Bring music / earplugs

Review

(1) Are the vectors linearly dependent? If so, write one as a linear combination of the others.

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 11 \\ 1 \end{bmatrix}$$

Let $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$

1 solution
$c_1 = c_2 = c_3 = 0$
$\boxed{\text{NO}}$
Lin. independent

∞-many solutions
$c_1 = c_2 = c_3 = 0$ and other solutions
$\boxed{\text{YES}}$
Lin. dependent

$$\begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ -1 & 3 & 11 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right]$$

$$\rightsquigarrow \begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \text{RREF}$$

$\boxed{c_3 = t}$

Yes, linearly dependent.

$c_1 - 2c_3 = 0 \rightarrow \boxed{c_1 = 2t}$

$\boxed{c_2 = -3t}$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} t$$

Choose any nonzero t: e.g. $t = 1$

$c_1 = 2 \quad c_2 = -3 \quad c_3 = 1$

$\hookrightarrow \quad c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$

$2\vec{v}_1 - 3\vec{v}_2 + \vec{v}_3 = \vec{0}$

$$\boxed{\vec{v}_3 = -2\vec{v}_1 + 3\vec{v}_2}$$

② Write A^{-1} and A as a product of elementary matrices

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$R_2 - 3R_1 \quad \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} \quad E_1 = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \quad E_1^{-1} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

$(R_2 + 3R_1)$

$\frac{R_2}{-2} \quad \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad E_2 = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} \quad E_2^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$

$(-2R_2)$

$R_1 - 2R_2 \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad E_3 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \quad E_3^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$(R_1 + 2R_2)$

$$\underbrace{E_3 E_2 E_1}_{A^{-1}} A = I$$

$$\boxed{A^{-1} = E_3 E_2 E_1}$$

$$A = (A^{-1})^{-1}$$

$$A = (E_3 E_2 E_1)^{-1}$$

$$\boxed{A = E_1^{-1} E_2^{-1} E_3^{-1}}$$

Socks and shoes

(3) Find the general form of

$$\operatorname{span}\left(\begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \right)$$

$$c_1 \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Get conditions on a, b, c, d

$$\begin{array}{ccc} c_1 & c_2 & \\ \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 1 & a \\ 0 & 2 & b \\ 3 & 3 & c \\ 0 & 4 & d \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & 1 & a \\ 0 & 1 & b/2 \\ 0 & 0 & c-3a \\ 0 & 0 & d-2b \end{array}\right] \quad \text{REF/RREF}$$

One condition for each zero row

$$\boxed{\begin{array}{c} c-3a=0 \\ c=3a \end{array}} \qquad \boxed{\begin{array}{c} d-2b=0 \\ d=2b \end{array}}$$

$$\left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ such that } c=3a \text{ and } d=2b \right\}$$ ✓

$$= \left\{ \begin{bmatrix} a & b \\ 3a & 2b \end{bmatrix} \right\}$$ ✓

ASIDE

4a) Find conditions on a, b, c, d so that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ commute

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} a & 0 \\ c & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$$

$b = 0$ and $c = 0$

b) Give an example where $AB = AC$ and $B \neq C$

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 9 & 9 \\ 9 & 9 \end{bmatrix}$$

$$\boxed{\begin{matrix} AB = AC \\ B \neq C \end{matrix}}$$

c) $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$

Find B, C so that $AB = AC$ and $B \neq C$

$$\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$